文章编号:1000 - 2367(2017)01 - 0098 - 08　　　DOI:10. 16366/j. cnki. 1000 - 2367. 2017. 01. 016

带时滞的 Holling-Tanner 比率依赖型捕食被捕食模型的 triple-zero 分支

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摘　要:主要讨论了一类被捕食者带扰动项且具有常数率收获的时滞捕食被捕食系统的 triple zero 分支问题. 首先得到了系统的平衡点是 triple zero 奇点的存在条件,随后将原系统开拆标准型的计算转化为一个新系统的四重零分支标准型问题,通过推广应用时滞微分方程的中心流形定理和标准型约化理论,推导出了原系统的 triple zero 分支的开拆标准型.

关键词:捕食被捕食;时滞;triple zero 分支;开拆标准型

中图分类号:O175. 1　　　**文献标志码**:A

捕食被捕食模型的鞍 - 结点分支、Hopf 分支、全局稳定性等性质已得到了较多的研究(见文献[1 -4]),但是对时滞捕食系统的 triple zero 分支问题的研究较少. 文献[5]已分析了系统 $\dot{x} = F(x(t),x(t-1),\bar{\alpha})$ 在原点处的三重零奇点分支问题,其中 $\bar{\alpha}\in\mathbf{R}^3$ 是一个参数向量,$x\in\mathbf{R}^n$,$F(0,\bar{\alpha})=0$,即对于所有原点附近的 $\bar{\alpha}$,通过使用中心流形定理和标准型理论,已推导出了具体的开拆标准型(三维常微分方程). 若文献[6] 中的系统不满足 $F(0,\bar{\alpha})=0$, 则文献[5]所推导的公式将不能直接应用,文献[6]给出了所提出系统的 Bogdanvo-Takens(B-T)分支标准型的计算. 文献[7]研究了带有非单调功能反应函数的时滞 Leslie-Gower 捕食被捕食模型的 B-T 分支问题,通过把原系统的 B-T 分支问题转化为一个新系统的三重零分支问题,推导出了原系统的 B-T 分支标准型. 这些已有的工作基础启发我们考虑如下系统

$$\dot{x} = rx\left(1-\frac{x}{K}\right)-\frac{\alpha y(x-\bar{m})}{Ay+x-\bar{m}}-\bar{h},\quad \dot{y} = sy\left(1-\frac{by(t-\dot{\tau})}{x(t-\dot{\tau})-\bar{m}}\right) \tag{1}$$

的 triple zero 奇点的存在性以及相应的开拆标准型. x 和 y 分别表示被捕食者和捕食者的种群密度,参数 r,$K,\alpha,A,\bar{m},\bar{h},s,b,\dot{\tau}$ 均为正常数,其生物学意义可以参看文献[8 - 10]. 当 $\bar{m}=\bar{h}=\dot{\tau}=0$,文献[8] 研究了系统(1) 的全局稳定性和极限环的唯一性. 当 $\dot{\tau}=0$ 时,文献[9]分析了系统(1)各种依赖于原始参数的分支现象,如 Hopf 分支、后向分支、B-T 分支等. 文献[10]主要讨论了系统(1)的 B - T 分支.

1　triple zero 分支的存在性

接下来,开始讨论系统(1)的 triple zero 分支存在性. 为计算简便,首先令 $X=x-\bar{m}$,$Y=y$,则系统(1) 可以化为(仍然用 x,y 来分别表示 X,Y)

$$\dot{x} = r(x+\bar{m})\left(1-\frac{x+\bar{m}}{K}\right)-\frac{\alpha xy}{Ay+x}-\bar{h},\quad \dot{y} = sy\left(1-\frac{by(t-\dot{\tau})}{x(t-\dot{\tau})}\right). \tag{2}$$

令 $\tau = r\dot{\tau}$,$\bar{t}=rt$,$X(\bar{t}) = \dfrac{x(t)}{K}$,$Y(\bar{t}) = \dfrac{\alpha y(t)}{rK}$,则系统(2) 可化为下面的时滞微分动力系统(仍然用 x,y,t 来

收稿日期:2016-05-04;修回日期:2016-12-13.

基金项目:国家自然科学基金(11601131;11501177);国家留学基金资助;河南省教育厅科学技术研究重点项目(17A110025;15A110034).

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分别表示 $X,Y,\bar{t}$)

$$\dot{x} = (x+m)(1-x-m) - \frac{xy}{ay+x} - h,\quad \dot{y} = \delta y\left(\beta - \frac{y(t-\tau)}{x(t-\tau)}\right), \tag{3}$$

其中 $m = \frac{\bar{m}}{K}, a = \frac{Ar}{\alpha}, \delta = \frac{s\bar{h}}{\alpha}, \beta = \frac{\alpha}{br}, h = \frac{\bar{h}}{r}$.

经计算知当

(H1) $0 < m < \frac{1}{2}\left(1 - \frac{\beta}{a\beta+1}\right), h = h_0 = \frac{1}{4}\left(\frac{\beta}{a\beta+1} - 1\right)^2 + \frac{m\beta}{a\beta+1}$ 成立时, $P_* = (x_*, y_*)$ 是系统(3)的唯一一个内部正平衡点,其中 $x_* = -\frac{1}{2}\left(\frac{\beta}{a\beta+1} + 2m - 1\right), y_* = \beta x_*$. 然后将系统(3)在平衡点 P_* 处进行线性化,可得如下线性系统

$$\dot{x} = a_{11}x(t) + a_{12}y(t),\quad \dot{y} = \delta\beta^2 x(t-\tau) - \delta\beta y(t-\tau), \tag{4}$$

其中 $a_{11} = \frac{\beta}{(a\beta+1)^2}, a_{12} = -\frac{1}{(a\beta+1)^2}$, 且系统(4)对应的特征方程为 $F(\lambda) = \lambda^2 + \left[\delta\beta e^{-\lambda\tau} - \frac{\beta}{(a\beta+1)^2}\right]\lambda = 0$.

明显地,如果 $F(0) = 0, F'(0) = 0, F''(0) \neq 0$ 成立,即 $\delta = \frac{1}{(a\beta+1)^2}, \tau \neq \frac{(a\beta+1)^2}{\beta}$,则 $\lambda = 0$ 是二重特征值(与其对应的 B-T 分支已经在文献[10]中被研究过);如果 $F(0) = 0, F'(0) = 0, F''(0) = 0, F'''(0) \neq 0$ 成立,即

(H2) $\delta = \delta_0 = \frac{1}{(a\beta+1)^2}, \tau = \tau_0 = \frac{(a\beta+1)^2}{\beta}$,

则 $\lambda = 0$ 是三重特征值.

当 $\tau \neq 0$ 时,将 $\lambda = \xi + \omega i (\omega \neq 0)$ 代入方程 $F(\lambda) = 0$ 且分离实部和虚部,消去方程组中的三角函数可得

$$[(a\beta+1)^4\xi^2 - 2\beta(a\beta+1)^2\xi + (a\beta+1)^4\omega^2 + \beta^2]e^{2\xi\tau} - \beta^2 = 0. \tag{5}$$

因为 a,β,ω 均为正常数,因此当 $\xi = 0$ 时,得 $(a\beta+1)^4\omega^2 \neq 0$,与方程(5)矛盾. 所以特征方程没有纯虚根. 故得如下定理.

定理 1　若(H1)和(H2)成立,则系统(3)在平衡点 P_* 处经历 triple zero 分支.

2　Triple zero 分支规范型

首先,通过变换 $t = \tau t$,系统(3)可化简为(仍然用 t 来表示 t)

$$\dot{x} = \tau\left((x+m)(1-x-m) - \frac{xy}{ay+x} - h\right),\quad \dot{y} = \tau\delta y\left(\beta - \frac{y(t-1)}{x(t-1)}\right). \tag{6}$$

然后开始计算系统(6)在正平衡点 P_* 处的开拆标准型. 令 $h = h_0 + \mu_1, \tau = \tau_0 + \mu_2, \delta = \delta_0 + \mu_3$ 其中 $\mu = (\mu_1, \mu_2, \mu_3)$ 充分小,则系统(6)变为

$$\begin{cases} \dot{x} = (\tau_0 + \mu_2)\left((x+m)(1-x-m) - \frac{xy}{ay+x} - h_0 - \mu_1\right), \\ \dot{y} = (\tau_0 + \mu_2)(\delta_0 + \mu_3)y\left(\beta - \frac{y(t-1)}{x(t-1)}\right). \end{cases} \tag{7}$$

显然,由于 μ_1 的存在, P_* 不再是系统(7)的平衡点. 因此,计算系统(6)在 P_* 处的规范型等价于计算系统

$$\begin{cases} (7), \\ \dot{\mu}_1 = 0 \end{cases} \tag{8}$$

在 $(x_*, y_*, 0)$ 处的四重零根分支的规范型. 接下来把参数 μ_1 和变量 x,y 同等看待. 对于系统(8),令 $X = x - x_*, Y = y - y_*, \mu_1 = \mu_1$,并且把 X,Y 仍记为 x,y ,然后泰勒展开得

$$\dot{z}_t = L_0(z_t) + L_1(\mu_2,\mu_3)z_t + F(z_t,\mu_2,\mu_3), \tag{9}$$

其中 $z_t = (x(t),y(t),\mu_1)^{\mathrm{T}}, z(t+\theta) = \varphi(\theta), \varphi = (\varphi_1,\varphi_2,\mu_1)^{\mathrm{T}} \in C_3$,

$$L_0(\varphi) = \tau_0\begin{pmatrix} a_{11}\varphi_1(0) + a_{12}\varphi_2(0) - \mu_1 \\ \delta_0\beta^2\varphi_1(-1) - \delta_0\beta\varphi_2(-1) \\ 0 \end{pmatrix},$$

$$L_1(\mu_2,\mu_3)\varphi = \begin{pmatrix} \mu_2(a_{11}\varphi_1(0) + a_{12}\varphi_2(0) - \mu_1) \\ \mu_2(\delta_0\beta^2\varphi_1(-1) - \delta_0\beta\varphi_2(-1)) + \\ \tau_0\mu_3(\beta^2\varphi_1(-1) - \beta\varphi_2(-1)) \\ 0 \end{pmatrix},$$

$$F(\varphi,\mu_2,\mu_3) = \begin{pmatrix} \tau_0(k_1\varphi_1^2(0) + k_2\varphi_1(0)\varphi_2(0) + k_3\varphi_2^2(0)) + h.o.t. \\ \tau_0\delta_0[k_4\varphi_1^2(-1) + k_5\varphi_1(-1)\varphi_2(0) + k_6\varphi_1(-1)\cdot \\ \varphi_2(-1) + k_7\varphi_2(-1)\varphi_2(0)] + h.o.t. \\ 0 \end{pmatrix},$$

$$k_1 = -1 - \frac{2a\beta^2}{(a\beta+1)^2[(2m-1)(a\beta+1)+\beta]},$$

$$k_2 = \frac{4a\beta}{(a\beta+1)^2[(2m-1)(a\beta+1)+\beta]},$$

$$k_3 = -\frac{2a}{(a\beta+1)^2[(2m-1)(a\beta+1)+\beta]},$$

$$k_4 = \frac{2\beta^2(a\beta+1)}{(2m-1)(a\beta+1)+\beta},$$

$$k_5, k_6 = -\frac{2\beta(a\beta+1)}{(2m-1)(a\beta+1)+\beta},$$

$$k_7 = \frac{2(a\beta+1)}{(2m-1)(a\beta+1)+\beta}.$$

系统(9)的在零解处的线性系统为 $z(t) = L_0(z_t)$，且其对应线性系统的特征矩阵为

$$\Delta(\lambda) = \begin{pmatrix} \lambda - \tau_0 a_{11} & -\tau_0 a_{12} & \tau_0 \\ -\tau_0\delta_0\beta^2 \mathrm{e}^{-\lambda} & \lambda + \tau_0\delta_0\beta\mathrm{e}^{-\lambda} & 0 \\ 0 & 0 & \lambda \end{pmatrix}. \tag{10}$$

然后,根据文献[5,7]中的方法,可以得到如下引理.

引理1 P 和它的对偶空间 P^* 的基有如下的表达式: $P = \mathrm{span}\Phi, \Phi(\theta) = (\varphi_1(\theta),\varphi_2(\theta),\varphi_3(\theta),\varphi_4(\theta))$, $\mathrm{P}^* = \mathrm{span}\Psi, \Psi(s) = (\psi_1(s),\psi_2(s),\psi_3(s),\psi_4(s))$, 其中 $\varphi_1(\theta) = u_1 \in \mathbf{R}^n/\{0\}, \varphi_2(\theta) = u_2 + \theta u_1, \varphi_3(\theta) = u_3 + \theta u_2 + \frac{\theta^2}{2!}u_1, \varphi_4(\theta) = u_4 + \theta u_3 + \frac{\theta^2}{2!}u_2 + \frac{\theta^3}{3!}u_1, u_1,u_2,u_3,u_4 \in \mathbf{R}^n, \psi_1(s) = v_1 - sv_2 + \frac{s^2}{2!}v_3 - \frac{s^3}{3!}v_4, \psi_2(s) = v_2 - sv_3 + \frac{s^2}{2!}v_4, \psi_3(s) = v_3 - sv_4, \psi_4(s) = v_4 \in \mathbf{R}^{n*}\backslash\{0\}, v_1,v_2,v_3,v_4 \in \mathbf{R}^{n*}$ 满足

(1) $\Delta(0)u_1 = 0$, (2) $\Delta(0)u_2 + \Delta'(0)u_1 = 0$, (3) $\Delta(0)u_3 + \Delta'(0)u_2 + \frac{1}{2!}\Delta''(0)u_1 = 0$,

(4) $\Delta(0)u_4 + \Delta'(0)u_3 + \frac{1}{2!}\Delta''(0)u_2 + \frac{1}{3!}\Delta'''(0)u_1 = 0$, (5) $\Delta^{\mathrm{T}}(0)v_4^{\mathrm{T}} = 0$,

(6) $\Delta^{\mathrm{T}}(0)v_3^{\mathrm{T}} + \Delta'^{\mathrm{T}}(0)v_4^{\mathrm{T}} = 0$, (7) $\Delta^{\mathrm{T}}(0)v_2^{\mathrm{T}} + \Delta'^{\mathrm{T}}(0)v_3^{\mathrm{T}} + \frac{1}{2!}\Delta''^{\mathrm{T}}(0)v_4^{\mathrm{T}} = 0$,

(8) $\Delta^{\mathrm{T}}(0)v_1^{\mathrm{T}} + \Delta'^{\mathrm{T}}(0)v_2^{\mathrm{T}} + \frac{1}{2!}\Delta''^{\mathrm{T}}(0)v_3^{\mathrm{T}} + \frac{1}{3!}\Delta'''^{\mathrm{T}}(0)v_4^{\mathrm{T}} = 0$,

(9) $v_4(B_0+I_0)u_4-\frac{1}{2!}v_4B_0\varphi_3+\frac{1}{3!}v_4B_0u_2-\frac{1}{4!}v_4B_0u_1=1$,

(10) $v_3(B_0+I_0)u_4-\frac{1}{2!}v_3B_0u_3+\frac{1}{3!}v_3B_0u_2-\frac{1}{4!}v_3B_0u_1=\frac{1}{2!}v_4B_0u_4-\frac{1}{3!}v_4B_0u_3+\frac{1}{4!}v_4B_0u_2-\frac{1}{5!}v_4B_0\varphi_1$,

(11) $v_2(B_0+I_0)u_4-\frac{1}{2!}v_2B_0u_3+\frac{1}{3!}v_2B_0u_2-\frac{1}{4!}v_2B_0u_1-\frac{1}{2!}v_3B_0u_4+\frac{1}{3!}v_3B_0u_3-\frac{1}{4!}v_3B_0u_2+\frac{1}{5!}v_3B_0u_1+\frac{1}{3!}v_4B_0u_4-\frac{1}{4!}v_4B_0u_3+\frac{1}{5!}v_4B_0u_2-\frac{1}{6!}v_4B_0u_1=0$,

(12) $v_1(B_0+I_0)u_4-\frac{1}{2!}v_1B_0u_3+\frac{1}{3!}v_1B_0u_2-\frac{1}{4!}v_1B_0u_1-\frac{1}{2!}v_2B_0u_4+\frac{1}{3!}v_2B_0u_3-\frac{1}{4!}v_2B_0u_2+\frac{1}{5!}v_2B_0u_1+\frac{1}{3!}v_3B_0u_4-\frac{1}{4!}v_3B_0u_3+\frac{1}{5!}v_3B_0u_2-\frac{1}{6!}v_3B_0u_1-\frac{1}{4!}v_4B_0u_4+\frac{1}{5!}v_4B_0u_3-\frac{1}{6!}v_4B_0u_2+\frac{1}{7!}v_4B_0u_1=0$,其中 I_0 为三阶单位矩阵,$B_0=\begin{pmatrix}0&0&0\\ \beta&-1&0\\ 0&0&0\end{pmatrix}$.

由引理 1 得

$$\Phi(\theta)=\begin{pmatrix}1&\theta&1+\frac{\theta^2}{2}&\theta+\frac{\theta^3}{6}\\ \beta&(\theta-1)\beta&\frac{\beta}{2}(\theta^2-2\theta+2)&\frac{\beta}{6}(\theta^3-3\theta^2+6\theta-3)\\ 0&0&0&-\frac{\beta}{2(a\beta+1)^2}\end{pmatrix},$$

$$\Psi(0)=\begin{pmatrix}-\frac{17}{18}&\frac{35}{18\beta}&\frac{181(a\beta+1)^2}{270\beta}\\ \frac{2}{3}&-\frac{2}{3\beta}&\frac{17(a\beta+1)^2}{18\beta}\\ 2&-\frac{2}{\beta}&-\frac{2(a\beta+1)^2}{3\beta}\\ 0&0&-\frac{2(a\beta+1)^2}{\beta}\end{pmatrix}. \tag{11}$$

令 $\tilde{x}=(z_1\quad z_2\quad z_3\quad \tilde{\mu})^T$ 且 $\tilde{\mu}=\frac{\mu_1}{u_{43}}, z_t=\Phi\tilde{x}, \tilde{x}\in\mathbf{R}^4$,则系统(9)可以分解成

$$\dot{\tilde{x}}=B\tilde{x}+\Psi(0)[L_1(\mu_2,\mu_3)(\Phi\tilde{x}+\tilde{y})+F(\Phi\tilde{x}+\tilde{y},\mu_2,\mu_3)], \tag{12}$$

系统(12)的泰勒展开式为

$$\Psi(0)[L_1(\mu_2,\mu_3)(\Phi\tilde{x}+\tilde{y})+F(\Phi\tilde{x}+\tilde{y},\mu_2,\mu_3)]=\sum_{j\geq 2}\frac{1}{j!}f_j^1(\tilde{x},\tilde{y},\mu_2,\mu_3), \tag{13}$$

其中 $f_j^i(i=1,2)$ 表示度为 j,变量为 $(\tilde{x},\tilde{y},\mu_2,\mu_3)$,且系数属于 $\mathbf{R}^4\times\ker\pi$ 的齐次多项式. 用 $V_j^6(\mathbf{R}^4)$ 来表示有 6 个实变量 $z_1,z_2,z_3,\tilde{\mu},\mu_2,\mu_3$,且度为 j 的齐次多项式的线性空间. 对于 $j\geq 2$,M_j^1 表示定义域为 $V_j^6(\mathbf{R}^4)$ 的算子,其值域也在同一空间:

$$M_j^1(p))(\tilde{x},\mu_2,\mu_3)=D_zp(\tilde{x},\mu_2,\mu_3)B\tilde{x}-Bp(\tilde{x},\mu_2,\mu_3), p(\tilde{x},\mu_2,\mu_3)\in V_j^6(\mathbf{R}^4). \tag{14}$$

应用文献[11]中的变量的连续变换理论,系统(12)可化为如下的规范型

$$\dot{\tilde{x}}=B\tilde{x}+\frac{1}{2!}g_2^1(\tilde{x},0,\mu_2,\mu_3)+\frac{1}{3!}g_3^1(\tilde{x},0,\mu_2,\mu_3)+h.o.t., \tag{15}$$

其中 $g_j^1=(I-P_{I,j}^1)\tilde{f}_j^1(\tilde{x},0,\mu_2,\mu_3)\in(\mathrm{Im}M_j^1)^c$,且 $\tilde{f}_j^1$ 表示在以前变量变换后的阶为 j 的项,$P_{I,j}^1\tilde{f}_j^1$ 表示将 $\tilde{f}_j^1$ 映射到算子 M_j^1 的相空间 $\mathrm{Im}M_j^1$.

由(14) 式可知 $V_2^6\begin{pmatrix}R^3\\ 0\end{pmatrix}$ 在 M_2^1 上的象的基为

$$\begin{pmatrix}z_1z_2\\0\\0\\0\end{pmatrix},\begin{pmatrix}z_2z_3\\0\\0\\0\end{pmatrix},\begin{pmatrix}z_3\tilde{\mu}\\0\\0\\0\end{pmatrix},\begin{pmatrix}z_2^2+z_1z_3\\0\\0\\0\end{pmatrix},\begin{pmatrix}z_2z_3+z_1\tilde{\mu}\\0\\0\\0\end{pmatrix},\begin{pmatrix}z_2\tilde{\mu}\\0\\0\\0\end{pmatrix},\begin{pmatrix}z_2\mu_2\\0\\0\\0\end{pmatrix},\begin{pmatrix}z_2\mu_3\\0\\0\\0\end{pmatrix},\begin{pmatrix}z_3^2+z_2\tilde{\mu}\\0\\0\\0\end{pmatrix},\begin{pmatrix}z_3\tilde{\mu}\\0\\0\\0\end{pmatrix},\begin{pmatrix}z_3\mu_2\\0\\0\\0\end{pmatrix},\begin{pmatrix}z_3\mu_3\\0\\0\\0\end{pmatrix},\begin{pmatrix}\tilde{\mu}^2\\0\\0\\0\end{pmatrix},$$

$$\begin{pmatrix}\tilde{\mu}\mu_2\\0\\0\\0\end{pmatrix},\begin{pmatrix}\tilde{\mu}\mu_3\\0\\0\\0\end{pmatrix},\begin{pmatrix}-z_1^2\\2z_1z_2\\0\\0\end{pmatrix},\begin{pmatrix}-z_2^2\\2z_2z_3\\0\\0\end{pmatrix},\begin{pmatrix}-z_3^2\\2z_3\tilde{\mu}\\0\\0\end{pmatrix},\begin{pmatrix}\mu_2^2\\0\\0\\0\end{pmatrix},\begin{pmatrix}\mu_3^2\\0\\0\\0\end{pmatrix},\begin{pmatrix}-z_1z_2\\z_2^2+z_1z_3\\0\\0\end{pmatrix},\begin{pmatrix}-z_1z_3\\z_2z_3+z_1\tilde{\mu}\\0\\0\end{pmatrix},\begin{pmatrix}-z_1\tilde{\mu}\\z_2\tilde{\mu}\\0\\0\end{pmatrix},\begin{pmatrix}-z_1\mu_2\\z_2\mu_2\\0\\0\end{pmatrix},\begin{pmatrix}-z_1\mu_3\\z_2\mu_3\\0\\0\end{pmatrix},$$

$$\begin{pmatrix}-z_2z_3\\z_3^2+z_2\tilde{\mu}\\0\\0\end{pmatrix},\begin{pmatrix}-z_2\tilde{\mu}\\z_3\tilde{\mu}\\0\\0\end{pmatrix},\begin{pmatrix}-z_2\mu_2\\z_3\mu_2\\0\\0\end{pmatrix},\begin{pmatrix}-z_2\mu_3\\z_3\mu_3\\0\\0\end{pmatrix},\begin{pmatrix}-z_3\tilde{\mu}\\\tilde{\mu}^2\\0\\0\end{pmatrix},\begin{pmatrix}-z_3\mu_2\\\tilde{\mu}\mu_2\\0\\0\end{pmatrix},\begin{pmatrix}-z_3\mu_3\\\tilde{\mu}\mu_3\\0\\0\end{pmatrix},\begin{pmatrix}\mu_2\mu_3\\0\\0\\0\end{pmatrix},\begin{pmatrix}0\\-z_1^2\\2z_1z_2\\0\end{pmatrix},\begin{pmatrix}0\\-z_2^2\\2z_2z_3\\0\end{pmatrix},\begin{pmatrix}0\\-z_3^2\\2z_3\tilde{\mu}\\0\end{pmatrix},\begin{pmatrix}0\\\tilde{\mu}^2\\0\\0\end{pmatrix},$$

$$\begin{pmatrix}0\\\mu_2^2\\0\\0\end{pmatrix},\begin{pmatrix}0\\\mu_3^2\\0\\0\end{pmatrix},\begin{pmatrix}0\\-z_1z_2\\z_2^2+z_1z_3\\0\end{pmatrix},\begin{pmatrix}0\\-z_1\tilde{\mu}\\z_2\tilde{\mu}\\0\end{pmatrix},\begin{pmatrix}0\\-z_1\mu_2\\z_2\mu_2\\0\end{pmatrix},\begin{pmatrix}0\\-z_1\mu_3\\z_2\mu_3\\0\end{pmatrix},\begin{pmatrix}0\\-z_2z_3\\z_3^2+z_2\tilde{\mu}\\0\end{pmatrix},\begin{pmatrix}0\\-z_2\tilde{\mu}\\z_3\tilde{\mu}\\0\end{pmatrix},\begin{pmatrix}0\\-z_2\mu_2\\z_3\mu_2\\0\end{pmatrix},\begin{pmatrix}0\\-z_2\mu_3\\z_3\mu_3\\0\end{pmatrix},\begin{pmatrix}0\\-z_3\tilde{\mu}\\\tilde{\mu}^2\\0\end{pmatrix},$$

$$\begin{pmatrix}0\\-z_3\mu_2\\\tilde{\mu}\mu_2\\0\end{pmatrix},\begin{pmatrix}0\\-z_3\mu_3\\\tilde{\mu}\mu_3\\0\end{pmatrix},\begin{pmatrix}0\\\tilde{\mu}\mu_2\\0\\0\end{pmatrix},\begin{pmatrix}0\\\tilde{\mu}\mu_3\\0\\0\end{pmatrix},\begin{pmatrix}0\\\mu_2\mu_3\\0\\0\end{pmatrix}.$$

$(M_2{}^1)^c$ 的基为

$$\begin{pmatrix}0\\0\\z_1^2\\0\end{pmatrix},\begin{pmatrix}0\\0\\z_2^2\\0\end{pmatrix},\begin{pmatrix}0\\0\\\mu_2^2\\0\end{pmatrix},\begin{pmatrix}0\\0\\\mu_3^2\\0\end{pmatrix},\begin{pmatrix}0\\0\\z_1z_2\\0\end{pmatrix},\begin{pmatrix}0\\0\\z_1\tilde{\mu}\\0\end{pmatrix},\begin{pmatrix}0\\0\\z_1\mu_2\\0\end{pmatrix},\begin{pmatrix}0\\0\\z_1\mu_3\\0\end{pmatrix},\begin{pmatrix}0\\0\\z_1z_3\\0\end{pmatrix},\begin{pmatrix}0\\0\\z_2\tilde{\mu}\\0\end{pmatrix},\begin{pmatrix}0\\0\\z_2\mu_2\\0\end{pmatrix},\begin{pmatrix}0\\0\\z_2\mu_3\\0\end{pmatrix},\begin{pmatrix}0\\0\\z_3\mu_2\\0\end{pmatrix},\begin{pmatrix}0\\0\\z_3\mu_3\\0\end{pmatrix},\begin{pmatrix}0\\0\\\mu_2\mu_3\\0\end{pmatrix}.$$

对于任意的 $p(\tilde{x},\mu_2,\mu_3)\in V_2^6(\mathbf{R}^4)$,令 $Gp=\mathrm{Proj}_{(M_2^1)^c}p$,则 Gp 的值如下

$$Gp=(I-P_{i,2}^1)p=\begin{cases}p, & p\in \mathrm{Im}(M_2^1)^c,\\ 0, & p\in \mathrm{Im}(M_2^1),\end{cases}\quad G\begin{pmatrix}0\\0\\z_3^2\\0\end{pmatrix}=\begin{pmatrix}0\\0\\-\frac{4}{3}z_2\tilde{\mu}\\0\end{pmatrix},G\begin{pmatrix}z_1^2\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\2(z_2^2+z_1z_3)\\0\end{pmatrix},G\begin{pmatrix}0\\0\\z_2z_3\\0\end{pmatrix}=\begin{pmatrix}0\\0\\-\frac{1}{3}z_1\tilde{\mu}\\0\end{pmatrix},$$

$$G\begin{pmatrix}z_2^2\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\-\frac{2}{3}\tilde{\mu}z_2\\0\end{pmatrix},G\begin{pmatrix}z_1z_3\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\\frac{2}{3}\tilde{\mu}z_2\\0\end{pmatrix},G\begin{pmatrix}z_1\mu_2\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\z_3\mu_2\\0\end{pmatrix},G\begin{pmatrix}z_1\mu_3\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\z_3\mu_3\\0\end{pmatrix},G\begin{pmatrix}0\\z_1^2\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\2z_1z_2\\0\end{pmatrix},$$

$$G\begin{pmatrix}0\\z_2^2\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\-\frac{2}{3}z_1\tilde{\mu}\\0\end{pmatrix},G\begin{pmatrix}0\\z_1z_2\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\z_2^2+z_1z_3\\0\end{pmatrix},G\begin{pmatrix}0\\z_1z_3\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\\frac{2}{3}z_1\tilde{\mu}\\0\end{pmatrix},G\begin{pmatrix}0\\z_1\tilde{\mu}\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\z_2\tilde{\mu}\\0\end{pmatrix},G\begin{pmatrix}0\\z_1\mu_2\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\z_2\mu_2\\0\end{pmatrix},$$

$$G\begin{pmatrix}0\\z_1\mu_3\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\z_2\mu_3\\0\end{pmatrix},G\begin{pmatrix}0\\z_2z_3\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\-\frac{1}{3}z_2\tilde{\mu}\\0\end{pmatrix},G\begin{pmatrix}0\\z_2\mu_2\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\z_3\mu_2\\0\end{pmatrix},G\begin{pmatrix}0\\z_2\mu_3\\0\\0\end{pmatrix}=\begin{pmatrix}0\\0\\z_3\mu_3\\0\end{pmatrix}.$$

因为 $\varphi=\Phi(\theta)\tilde{x}$,由(11)式可得

$$\varphi_1(0)=z_1+z_3,\varphi_2(0)=\beta(z_1-z_2+z_3-\frac{1}{2}\tilde{\mu}),\varphi_1(-1)=z_1-z_2+\frac{3}{2}z_3-\frac{7}{6}\tilde{\mu},$$

$$\varphi_2(-1)=\beta(z_1-2z_2+\frac{5}{2}z_3-\frac{13}{6}\tilde{\mu}).$$

由(9)、(11)和(13)式可得

$$\frac{1}{2}f_2^1(\tilde{x},0,\mu_2,\mu_3)=\begin{pmatrix}\frac{\beta}{(a\beta+1)^2}z_2\mu_2+\frac{35}{18}(a\beta+1)^2z_2\mu_3-\frac{35}{18}\frac{\beta}{(a\beta+1)^2}z_3\mu_2-\\ \frac{35}{18}(a\beta+1)^2z_3\mu_3+\frac{\beta}{(a\beta+1)^2}\tilde{\mu}\mu_2+\frac{35}{18}(a\beta+1)^2\tilde{\mu}\mu_3\\ -\frac{2}{3}(a\beta+1)^2z_2\mu_3+\frac{2\beta}{3(a\beta+1)^2}z_3\mu_2+\frac{2}{3}(a\beta+1)^2z_3\mu_3-\frac{2}{3}(a\beta+1)^2\tilde{\mu}\mu_3\\ -2(a\beta+1)^2z_2\mu_3+\frac{2\beta}{(a\beta+1)^2}z_3\mu_2+2(a\beta+1)^2z_3\mu_3-2(a\beta+1)^2\tilde{\mu}\mu_3\\ 0\end{pmatrix}+$$

$$\begin{pmatrix}m_{10}z_1^2+2m_{10}z_1z_3+m_{11}z_2^2+m_{12}z_2z_3+m_{13}z_2\tilde{\mu}+m_{14}z_3{}^2+m_{15}z_3\tilde{\mu}+m_{16}\tilde{\mu}^2\\ m_{20}z_1^2+2m_{20}z_1z_3+m_{21}z_2^2+m_{22}z_2z_3+m_{23}z_2\tilde{\mu}+m_{24}z_3^2+m_{25}z_3\tilde{\mu}+m_{26}\tilde{\mu}^2\\ 3(m_{20}z_1^2+2m_{20}z_1z_3+m_{21}z_2^2+m_{22}z_2z_3+m_{23}z_2\tilde{\mu}+m_{24}z_3^2+m_{25}z_3\tilde{\mu}+m_{26}\tilde{\mu}^2)\\ 0\end{pmatrix},\tag{16}$$

其中 $m_{10}=\frac{17(a\beta+1)^2}{18\beta},m_{11}=\frac{17a\beta}{9p_0},m_{12}=\frac{35(a\beta+1)}{18p_0},m_{13}=-\frac{19a\beta+70}{27p_0},m_{15}=\frac{245(a\beta+1)}{54p_0},m_{16}=-\frac{229a\beta+280}{108p_0},m_{14}=\frac{(a\beta+1)[34(a\beta+1)^2m-17a^2\beta^2+17a\beta^2-34a\beta-18\beta-17]}{18\beta p_0},m_{20}=-\frac{12}{17}m_{10},m_{21}=-\frac{4a\beta}{3p_0},m_{22}=-\frac{2(a\beta+1)}{3p_0},m_{23}=-\frac{4(a\beta-2)}{9p_0},m_{24}=-\frac{2(a\beta+1)(2(a\beta+1)^2m-a^2\beta^2+a\beta^2-2a\beta-1)}{3\beta p_0},$ $m_{25}=-\frac{14(a\beta+1)}{9p_0},m_{26}=\frac{5a\beta+8}{9p_0},p_0=(2m-1)(a\beta+1)+\beta.$

因此,由(15)、(16)式可知 $\frac{1}{2!}g_2{}^1(\tilde{x},0,\mu_2,\mu_3)=(0,0,W_1z_1\tilde{\mu}+W_2z_2\tilde{\mu}+W_3z_2\mu_3+W_4z_3\mu_2+W_5z_3\mu_3+\gamma_1z_1^2+\gamma_2z_2^2+\gamma_3z_1z_2+\gamma_4z_1z_3,0)^{\mathrm{T}}$,其中 $W_1=\frac{4m_{20}}{3}-\frac{2m_{21}}{3}-\frac{m_{32}}{3},W_2=\frac{4m_{10}}{3}-\frac{2m_{11}}{3}-\frac{m_{22}}{3}+m_{33}-\frac{4m_{34}}{3},W_3=-2(a\beta+1)^2,$ $W_4=\frac{2\beta}{(a\beta+1)^2},W_5=\frac{4}{3}(a\beta+1)^2,\gamma_1=-\frac{2(a\beta+1)^2}{\beta},\gamma_2=2m_{10}+m_{31},\gamma_3=2m_{20},\gamma_4=-\frac{19(a\beta+1)^2}{9\beta}$. 因此由(15)式可得

$$\dot{z}_1=z_2,\quad\dot{z}_2=z_3,\quad\dot{z}_3=N_0+N_1z_1+N_2z_2+N_3z_3+\gamma_1z_1^2+\gamma_2z_2^2+\gamma_3z_1z_2+\gamma_4z_1z_3+h.o.t.,\tag{17}$$

其中 $N_0=\frac{1}{u_{43}}\mu_1,N_1=\frac{W_1}{u_{43}}\mu_1,N_2=\frac{W_2}{u_{43}}\mu_1+W_3\mu_3,N_3=W_4\mu_2+W_5\mu_3$.

由坐标变换 $z_1\to z_1-\frac{N_1}{2\gamma_1},z_2\to z_2,z_3\to z_3$,系统(17)变为

$$\dot{z}_1=z_2,\quad\dot{z}_2=z_3,\quad\dot{z}_3=\varepsilon_1+\varepsilon_2z_2+\varepsilon_3z_3+\gamma_1z_1^2+\gamma_2z_2^2+\gamma_3z_1z_2+\gamma_4z_1z_3+h.o.t.,\tag{18}$$

其中 $\varepsilon_1 = N_0 - \frac{N_1^2}{4\gamma_1} = -\frac{2(a\beta+1)^2\mu_1}{\beta} + \frac{2[8(a\beta+1)^3m+p_1]^2(a\beta+1)^2\mu_1^2}{81(2am\beta-a\beta+2m+\beta-1)^2\beta^3}$,

$\varepsilon_2 = N_2 - \frac{\gamma_3 N_1}{2\gamma_1} = -\frac{4(a\beta+1)^2[38(a\beta+1)^3m+p_2]}{9\beta^2(2am\beta-a\beta+2m+\beta-1)}\mu_1 - 2(a\beta+1)^2\mu_3$,

$\varepsilon_3 = N_3 - \frac{\gamma_4 N_1}{2\gamma_1} = -\frac{19[8(a\beta+1)^3m+p_3](a\beta+1)^2}{81\beta^2(2am\beta-a\beta+2m+\beta-1)}\mu_1 + \frac{2\beta}{(a\beta+1)^2}\mu_2 + \frac{4}{3}(a\beta+1)^2\mu_3$,

$p_1 = -4a^3\beta^3 + 4a^2\beta^3 - 12a^2\beta^2 + a\beta^2 - 12a\beta + \beta - 4$,

$p_2 = -19a^3\beta^3 + 19a^2\beta^3 - 57a^2\beta^2 + 13a\beta^2 - 57a\beta + 19\beta - 19$,

$p_3 = -4a^3\beta^3 + 4a^2\beta^3 - 12a^2\beta^2 + a\beta^2 - 12a\beta + \beta - 4$.

由文献[12],系统(18)可变为如下的双曲标准型

$$\dot{z}_1 = z_2,\quad \dot{z}_2 = z_3,\quad \dot{z}_3 = \varepsilon_1 + \varepsilon_2 z_2 + \varepsilon_3 z_3 + A_1 z_1 z_2 + A_2 z_1 z_3 - \frac{z_1^2}{2} + h.o.t., \tag{19}$$

其中 $A_1 = -\frac{\gamma_3}{2\gamma_1} = -\frac{1}{3},\quad A_2 = -\frac{\gamma_4}{2\gamma_1} = -\frac{19}{36}$.

容易计算出 $\left.\frac{\partial(\varepsilon_1,\varepsilon_2,\varepsilon_3}{\partial(\mu_1,\mu_2,\mu_3)}\right|_{\mu_i=0} = -8(a\beta+1)^2 \neq 0$,有以下的定理.

定理 2[12] 考虑系统(19),保持 $\varepsilon_3 > 0$(或 $\varepsilon_3 < 0$)成立.则对于临界值 $\varepsilon_1 = \varepsilon_2 = 0$,可得一个非退化的 B-T 分支.奇点是尖点类型,出现如下的分支曲线:

(1) 在原点处的鞍 - 结点分支曲线 $\boldsymbol{T}:\varepsilon_1 = 0$.

(2)在平衡点 $(-\sqrt{2\varepsilon_1},0,0)$(或$(\sqrt{2\varepsilon_1},0,0)$)处的下临界(或上临界)Hopf 分支曲线 H:

$\varepsilon_3 A_1 A_2 \varepsilon_1 - \varepsilon_3^2\varepsilon_2 + A_2^2\varepsilon_1 > 0$,

$\varepsilon_2 = \frac{O(\varepsilon_1) + \sqrt{2\varepsilon_1\varepsilon_3^6 + O(\varepsilon_1^2)}(\varepsilon_3 A_2 - 1)}{\varepsilon_3^4}$(或 $\varepsilon_2 = \frac{O(\varepsilon_1) - \sqrt{2\varepsilon_1\varepsilon_3^6 + O(\varepsilon_1^2)}(\varepsilon_3 A_2 - 1)}{\varepsilon_3^4}$.

(3)在平衡点 $(\sqrt{2\varepsilon_1},0,0)$(或$(-\sqrt{2\varepsilon_1},0,0)$)处的相斥(或吸引)同宿关联曲线 H_{om}:

$\varepsilon_3 A_1 A_2 \varepsilon_1 - \varepsilon_3^2\varepsilon_2 + A_2^2\varepsilon_1 > 0$,

$\varepsilon_2 = \frac{O(\varepsilon_1) + 5\sqrt{2\varepsilon_1\varepsilon_3^6 + O(\varepsilon_1^2)}(\varepsilon_3 A_2 - 1)}{7\varepsilon_3^4}$(或 $\varepsilon_2 = \frac{O(\varepsilon_1) - 5\sqrt{2\varepsilon_1\varepsilon_3^6 + O(\varepsilon_1^2)}(\varepsilon_3 A_2 - 1)}{7\varepsilon_3^4}$.

3 结束语

通过文中的分析,可以看出随着时滞捕食系统在其某个内部正平衡点处的特征方程的零特征根重数的增加,该系统会出现更多种类的分支现象.为了更好的了解捕食模型的性质,会进一步分析多时滞和余维数更高的模型的动力学性质.

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Triple zero Bifurcation of a Delayed Ratio-dependent Holling-Tanner Predator-prey System

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Abstract: In this paper, the triple zero bifurcation of a delayed predator prey system with prey refuge and constant rate harvesting is considered. Firstly, the existence conditions under which the interior equilibrium of the system is a triple zero singularity are obtained. Then, the computational problem of the unfolding normal form of original system is transformed to compute a quadruple zero bifurcation normal form of a new system, by generalizing and applying the center manifold theorem and normal form reduction theory of delay differential equations, the normal form of the triple zero bifurcation of the original system is derived.

Keywords: predator-prey; delay; triple zero bifurcation; unfolding normal form