

一类随机 SWEIA 艾滋病毒传播模型的动力学分析

马怡婷,张太雷,邓金超

(长安大学 理学院,西安 710064)

摘要:研究了一类具有随机效应的 SWEIA 艾滋病毒传播模型.首先,通过构造 Lyapunov 函数证明了确定性模型平衡点的全局渐近稳定性,利用停顿理论等方法证明了随机模型正解的全局存在唯一性与有界性;其次,分析了随机模型的解在相应确定性模型的无病平衡点与地方病平衡点附近的震荡行为,并得到了随机模型解的平均持续与灭绝性的充分条件;最后,通过数值模拟进一步显示了模型的动力学行为.

关键词:随机模型;Itô 公式;震荡行为;持久性;灭绝性

中图分类号:O175.1

文献标志码:A

文章编号:1000-2367(2024)02-0041-10

艾滋病(AIDS)是一种对人类健康危害极大的传染病,它由人类免疫缺陷病毒(HIV)引起,通过大规模破坏人体免疫系统最重要的 CD4-T 淋巴细胞,使免疫功能失控,从而丧失抵御疾病的能力.因此,人体易感染各种慢性疾病,引发并发症,并能导致恶性肿瘤,疾病的死亡率较高^[1].国内外的多个学者对 HIV/AIDS 的传播规律进行了深入研究^[2-3].SHOFIANAH 等^[4]讨论了具有垂直传播与治疗且具有两个潜伏期的 HIV/AIDS 传染病模型的最优控制问题,利用 Pontryagin 原理,得到了以最小的控制成本减少感染/症状亚群数量的最有效控制.MARSUDI 等^[5]研究了一类确定性的 HIV/AIDS 模型,得到了各类平衡点的全局稳定性,并对参数进行敏感性分析,结果表明:无症状感染者(艾滋病前期)与易感者的有效接触率对 HIV/AIDS 的传播影响最大.但是在现实中,由于环境多变,生物会受到各类随机因素的干扰,且大多数问题都具有不确定性,确定性的传染病模型很难做到对实际情况进行具体描述.因此,学者们开始重视随机数学模型^[6-9],并取得了很大进展^[10-12].KHAN 等^[13]考虑了具有随机扰动和时滞的冠状病毒流行模型,研究结果表明:布朗运动与噪声项对流行病传播的影响非常高,若噪声很大,疾病可能会减少或消失.HOU 等^[14]提出了一类随机 SIHR 的 COVID-19 流行模型,数值分析了传播速率、噪声强度等参数对疾病传播的影响,并得到结果:在忽视环境噪声影响的情况下,确定性模型的阈值水平被高估.文献[15]研究了确定性的 SWEIA 艾滋病毒传播模型:

$$\begin{cases} \frac{dS}{dt} = \Lambda - \beta_1 SW - \beta_2 SE - \beta_3 SI - \mu S, & \frac{dW}{dt} = \beta_1 SW + \beta_2 SE + \beta_3 SI - (\mu + \rho)W, \\ \frac{dE}{dt} = \rho W - (\mu + \sigma)E, & \frac{dI}{dt} = \sigma E - (\mu + \alpha + \gamma)I, & \frac{dA}{dt} = \gamma I - \mu A. \end{cases} \quad (1)$$

为更好地描述 HIV 传播的动力学行为,在文献[15]的基础上引入随机扰动因素,建立具有随机扰动的 SWEIA 艾滋病毒传播模型.

收稿日期:2022-08-24;**修回日期:**2022-09-29.

基金项目:陕西省自然科学基金计划(2022JM-023).

作者简介:马怡婷(1997-),女,陕西商洛人,长安大学硕士研究生,研究方向为生物数学,E-mail:myt20202023@163.com.

通信作者:张太雷(1980-),男,湖北松滋人,长安大学教授,博士,E-mail:tlzhang@chd.edu.cn.

引用本文:马怡婷,张太雷,邓金超.一类随机 SWEIA 艾滋病毒传播模型的动力学分析[J].河南师范大学学报(自然科学版),2024,52(2):41-50.(Ma Yiting,Zhang Tailei,Deng Jinchao.Dynamic analysis of an SWEIA HIV infection model with stochastic effects[J].Journal of Henan Normal University(Natural Science Edition),2024,52(2):41-50.DOI:10.16366/j.cnki.1000-2367.2022.08.24.0001.)

1 模型建立

在文献[15]中, $S(t), W(t), E(t), I(t), A(t)$ 分别表示 t 时刻的易感者、艾滋病毒窗口期感染者、艾滋病毒无症状潜伏期患者、艾滋病毒有症状期患者、艾滋病毒移除者的个体数量, Λ 表示新增人口数量, μ 表示自然死亡率, $\beta_1, \beta_2, \beta_3$ 分别表示易感者与 W, E, I 仓室中患者的接触率, ρ, σ, γ 分别表示感染者从 W 仓室到 E 仓室、从 E 仓室到 I 仓室以及从 I 仓室到 A 仓室的比率. 由文献[15]知, 模型(1)的最大正向不变集为 $\Omega = \{(S, W, E, I, A) \in \mathbf{R}_+^5; S + W + E + I + A \leq \frac{\Lambda}{\mu}\}$, 基本再生数为 $R_0 = \frac{\Lambda(\beta_1\omega_2\omega_3 + \beta_2\rho\omega_3 + \beta_3\rho\sigma)}{\mu\omega_1\omega_2\omega_3}$, 其中 $\omega_1 = \mu + \rho, \omega_2 = \mu + \sigma, \omega_3 = \mu + \alpha + \gamma$. 当 $R_0 \leq 1$ 时, 模型(1)仅存在无病平衡点 $P^0(\frac{\Lambda}{\mu}, 0, 0, 0, 0)$ 且局部渐近稳定; 当 $R_0 > 1$ 时, 模型(1)既存在无病平衡点, 还存在唯一的地方病平衡点 $P^*(S^*, W^*, E^*, I^*, A^*)$ 且局部渐近稳定. 下面通过 Lyapunov 函数方法证得两类平衡点的全局渐近稳定性.

定理 1 当 $R_0 < 1$ 时, 无病平衡点 P^0 全局渐近稳定; 当 $R_0 = 1$ 时, 无病平衡点 P^0 全局吸引.

证明 构造 Lyapunov 函数 $V_1(t) = \frac{\beta_1(\omega_2\omega_3 + \beta_2\rho\omega_3 + \beta_3\rho\sigma)}{\omega_1\omega_2\omega_3}W + \frac{\beta_2\omega_3 + \beta_3\sigma}{\omega_2\omega_3}E + \frac{\beta_3}{\omega_3}I$. $V_1(t)$ 关于模型(1)对 t 求导有

$$\frac{dV_1(t)}{dt} = \frac{\beta_1(\omega_2\omega_3 + \beta_2\rho\omega_3 + \beta_3\rho\sigma)}{\omega_1\omega_2\omega_3} \frac{dW}{dt} + \frac{\beta_2\omega_3 + \beta_3\sigma}{\omega_2\omega_3} \frac{dE}{dt} + \frac{\beta_3}{\omega_3} \frac{dI}{dt} \leq (\beta_1W + \beta_2E + \beta_3I)(R_0 - 1).$$

由 LaSalle 不变集原理知, Ω 内的任何轨线都趋于 P^0 , 所以无病平衡点 P^0 全局吸引.

定理 2 当 $R_0 > 1$ 时, 地方病平衡点 P^* 全局渐近稳定.

证明 构造 Lyapunov 函数

$$V_2(t) = (S - S^* - S^* \ln \frac{S}{S^*}) + (W - W^* - W^* \ln \frac{W}{W^*}) + \frac{\beta_2\omega_3 S^* + \beta_3 S^* \sigma}{\omega_2\omega_3} (E - E^* - E^* \ln \frac{E}{E^*}) + \frac{\beta_3 S^*}{\omega_3} (I - I^* - I^* \ln \frac{I}{I^*}).$$

当 $R_0 > 1$ 时, $V_2(t)$ 关于模型(1)对 t 求导得:

$$\begin{aligned} \frac{dV_2(t)}{dt} &= (1 - \frac{S^*}{S}) \frac{dS}{dt} + (1 - \frac{W^*}{W}) \frac{dW}{dt} + \frac{\beta_2\omega_3 S^* + \beta_3 S^* \sigma}{\omega_2\omega_3} (1 - \frac{E^*}{E}) \frac{dE}{dt} + \frac{\beta_3 S^*}{\omega_3} (1 - \frac{I^*}{I}) \frac{dI}{dt} = \\ &\mu(2 - \frac{S^*}{S} - \frac{S}{S^*}) + \beta_1 S^* W^* (2 - \frac{S^*}{S} - \frac{S}{S^*}) + \beta_2 S^* E^* (3 - \frac{S^*}{S} - \frac{WE^*}{W^* E} - \frac{SW^* E}{S^* WE^*}) + \\ &\beta_3 S^* I^* (4 - \frac{S^*}{S} - \frac{WE^*}{W^* E} - \frac{EI^*}{E^* I} - \frac{SW^* I}{S^* WI^*}). \end{aligned}$$

根据几何平均数和算术平均数之间的关系得:

$$\frac{S^*}{S} + \frac{S}{S^*} \geq 2, \frac{S^*}{S} + \frac{WE^*}{W^* E} + \frac{SW^* E}{S^* WE^*} \geq 3, \frac{S^*}{S} + \frac{WE^*}{W^* E} + \frac{EI^*}{E^* I} + \frac{SW^* I}{S^* WI^*} \geq 4,$$

所以当 $R_0 > 1$ 时, $\frac{dV_2(t)}{dt} \leq 0$, 因此地方病平衡点 P^* 全局渐近稳定.

由于实际问题会受到大量外界随机因素的干扰, 例如: 病毒的传播可能受到公共卫生教育、媒体宣传报道等因素的影响. 因此, 本文在文献[15]的基础上考虑随机因素, 建立如下随机模型:

$$\begin{cases} dS = [\Lambda - \beta_1 SW - \beta_2 SE - \beta_3 SI - \mu S]dt + \sigma_1 S(t)dB_1(t), \\ dW = [\beta_1 SW + \beta_2 SE + \beta_3 SI - (\mu + \rho)W]dt + \sigma_2 W(t)dB_2(t), \\ dE = [\rho W - (\mu + \sigma)E]dt + \sigma_3 E(t)dB_3(t), \\ dI = [\sigma E - (\mu + \alpha + \gamma)I]dt + \sigma_4 I(t)dB_4(t), \\ dA = [\gamma I - \mu A]dt + \sigma_5 A(t)dB_5(t), \end{cases} \quad (2)$$

其中 $B_i(t) (i=1,2,\dots,5)$ 是独立的布朗运动, $\sigma_i (i=1,2,\dots,5)$ 是其强度系数.

2 全局正解的存在与唯一性与有界性

定理 3 对任意给定的初值 $(S(0), W(0), E(0), I(0), A(0)) \in \mathbf{R}_+^5$, 随机模型(2) 存在唯一解 $(S(t), W(t), E(t), I(t), A(t)) (t \geq 0)$, 且该解以概率 1 位于 \mathbf{R}_+^5 中, 即 $(S(t), W(t), E(t), I(t), A(t)) \in \mathbf{R}_+^5 (t \geq 0)$, a.s..

证明 因为随机模型(2) 的解满足局部 Lipschitz 连续性条件, 因此, 对任意给的初值 $(S(0), W(0), E(0), I(0), A(0)) \in \mathbf{R}_+^5$, 随机模型(2) 存在唯一的局部解 $(S(t), W(t), E(t), I(t), A(t)), t \in [0, \tau_e)$, τ_e 是爆破时间. 要证明解的全局性, 只需证 $\tau_e = \infty$, a.s..

对任给的初值 $(S(0), W(0), E(0), I(0), A(0)) \in \mathbf{R}_+^5$, 则存在足够大的正整数 k_0 , 使得 $(S(0), W(0), E(0), I(0), A(0))$ 均在 $[\frac{1}{k_0}, k_0]$ 上. 对所有的正整数 $k \geq k_0$, 定义停时

$$\tau_k = \inf\{t \in [0, \tau_e] : \min\{S(t), W(t), E(t), I(t), A(t)\} \leq \frac{1}{k} \text{ 或者 } \max\{S(t), W(t), E(t), I(t), A(t)\} \geq k\}.$$

令 $\inf \emptyset = \infty$ (\emptyset 表示空集). 显然, 当 $k \rightarrow \infty$ 时, τ_k 单增. 设 $\tau_\infty = \lim_{k \rightarrow \infty} \tau_k$, 用反证法证明 $\tau_\infty = \infty$, a.s..

假设 $\tau_\infty < \infty$, 则存在常数 $T > 0$ 和 $\varepsilon \in (0, 1)$, 满足 $P(\tau_\infty \leq T) > \varepsilon$. 因此, 存在正数 $k_1 > k_0$, 使得对 $\forall k \geq k_1, P(\tau_\infty \leq T) \geq \varepsilon$.

定义函数 $V(S, W, E, I, A) = (S - c - c \ln \frac{S}{c}) + (W - 1 - \ln W) + (E - 1 - \ln E) + (I - 1 - \ln I) + (A - 1 - \ln A)$, 这里 $c = \min\{\frac{\mu}{\beta_1}, \frac{\mu}{\beta_2}, \frac{\mu + \alpha}{\beta_3}\}$. 对 $\forall x > 0, (x - 1 - \ln x) \geq 0$, 则 V 为非负函数. 由 Itô 公式得: $dV(S, W, E, I, A) = LVdt + \sigma_1(S - c)dB_1 + \sigma_2(W - 1)dB_2 + \sigma_3(E - 1)dB_3 + \sigma_4(I - 1)dB_4 + \sigma_5(A - 1)dB_5$. 其中 $LV \leq \Lambda + 4\mu + \mu c + \gamma + \sigma + \alpha + \frac{1}{2}(c\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + \sigma_5^2) \triangleq K$. 这里 K 是正常数, 则

$$dV(S, W, E, I, A) \leq Kdt + \sigma_1(S - c)dB_1 + \sigma_2(W - 1)dB_2 + \sigma_3(E - 1)dB_3 + \sigma_4(I - 1)dB_4 + \sigma_5(A - 1)dB_5.$$

对上式两边从 0 到 $\tau_k \wedge T$ 积分并取期望得:

$$EV(S(\tau_k \wedge T), W(\tau_k \wedge T), E(\tau_k \wedge T), I(\tau_k \wedge T), A(\tau_k \wedge T)) \leq V(S(0), W(0), E(0), I(0), A(0)) + KT.$$

令 $\Omega_k = \{\tau_k \leq T\}$, 对 $\forall k \geq k_1$, 有 $P(\Omega_k) \geq \varepsilon$. 对 $\forall \omega \in \Omega$, 由停时定义知 $S(\tau_k, \omega), W(\tau_k, \omega), E(\tau_k, \omega), I(\tau_k, \omega), A(\tau_k, \omega)$ 中至少有一个等于 k 或 $\frac{1}{k}$,

$$V(S(\tau_k, \omega), W(\tau_k, \omega), E(\tau_k, \omega), I(\tau_k, \omega), A(\tau_k, \omega)) \geq \min\{k - 1 - \ln k, \frac{1}{k} - 1 - \ln \frac{1}{k}\}. \quad (3)$$

所以 $V(S(0), W(0), E(0), I(0), A(0)) + KT \geq \varepsilon \min\{k - 1 - \ln k, \frac{1}{k} - 1 - \ln \frac{1}{k}\}$. 令 $k \rightarrow \infty$, 则有 $\infty > V(S(0), W(0), E(0), I(0), A(0)) + KT = \infty$ 与假设矛盾. 所以 $\tau_\infty = \infty$, a.s..

3 随机模型(2) 的解在无病平衡点附近的渐近行为

当确定性模型(1) 的 $R_0 < 1$ 时, 无病平衡点 P^0 稳定, 接下来讨论模型(2) 的解在 P^0 附近的渐近行为.

定理 4 当 $R_0 < 1$, 且满足 $\sigma_2^2 < 2\mu + \rho - 2 - \frac{\beta_1 \Lambda}{\mu}, \sigma_3^2 < 2\mu + \sigma - \rho - \frac{\beta_2 \Lambda}{\mu}, \sigma_4^2 < 2(\mu + \alpha) + \gamma - \sigma -$

$\frac{\beta_3 \Lambda}{\mu}, \sigma_5^2 < 2\mu - \gamma$, 那么对任意初值 $(S(0), W(0), E(0), I(0), A(0)) \in \mathbf{R}_+^5$, 随机模型(2) 的解有如下性质:

$$\limsup_{t \rightarrow \infty} \frac{1}{t} E \int_0^t [(S(\xi) - \frac{\Lambda}{\mu})^2 + W^2(\xi) + E^2(\xi) + I^2(\xi) + A^2(\xi)] d\xi \leq \frac{\Lambda^2 [\mu(2\sigma_1^2 + (\mu + \rho)^2 + \mu^2) + \Lambda(\beta_1 + \beta_2 + \beta_3)]}{2n_1\mu^3}. \text{其中:}$$

$$n_1 = \min\{2\mu, \mu + \frac{\rho}{2} - \frac{2 + \sigma_2^2}{2} - \frac{\beta_1\Lambda}{2\mu}, \mu + \frac{\sigma}{2} - \frac{\rho + \sigma_3^2}{2} - \frac{\beta_2\Lambda}{2\mu}, \mu + \alpha + \frac{\gamma}{2} - \frac{\sigma + \sigma_4^2}{2} - \frac{\beta_3\Lambda}{2\mu}, \mu - \frac{\gamma + \sigma_5^2}{2}\}.$$

证明 定义函数 $V_{11} = \frac{1}{2}(S - \frac{\Lambda}{\mu})^2, V_{12} = \frac{1}{2}E^2, V_{13} = \frac{1}{2}I^2, V_{14} = \frac{1}{2}A^2, V_{15} = \frac{1}{2}(S - \frac{\Lambda}{\mu} + W)^2$. 由 Itô 公式得:

$$dV_{11} \leq [-\mu(S - \frac{\Lambda}{\mu})^2 + \frac{\beta_1\Lambda}{2\mu}W^2 + \frac{\beta_2\Lambda}{2\mu}E^2 + \frac{\beta_3\Lambda}{2\mu}I^2 + \frac{1}{2}\sigma_1^2S^2 + \frac{\Lambda(\beta_1 + \beta_2 + \beta_3)}{2\mu}S^2]dt + \sigma_1S(S - \frac{\Lambda}{\mu})dB_1,$$

$$dV_{12} = [E(\rho W - (\mu + \sigma)E) + \frac{1}{2}\sigma_3^2E^2]dt + \sigma_3E^2dB_3 \leq (\frac{\rho}{2}W^2 - (\mu + \sigma - \frac{\rho + \sigma_3^2}{2})E^2)dt + \sigma_3E^2dB_3,$$

$$dV_{13} = [(I(\sigma E - (\mu + \alpha + \gamma)I) + \frac{1}{2}\sigma_4^2I^2]dt + \sigma_4I^2dB_4 \leq (\frac{\sigma}{2}E^2 - (\mu + \alpha + \gamma - \frac{\sigma + \sigma_4^2}{2})I^2)dt + \sigma_4I^2dB_4,$$

$$dV_{14} = [A(\gamma I - \mu A) + \frac{1}{2}\sigma_5^2A^2]dt + \sigma_5A^2dB_5 \leq (\frac{\gamma}{2}I^2 - (\mu - \frac{\gamma + \sigma_5^2}{2})A^2)dt + \sigma_5A^2dB_5,$$

$$dV_{15} \leq [-\mu(S - \frac{\Lambda}{\mu})^2 - (\mu + \rho - \frac{2 + \sigma_2^2}{2})W^2 + \frac{\sigma_1^2\Lambda^2}{2\mu^2} + \frac{\Lambda^2(\mu + \rho)^2}{2\mu^2} + \frac{\Lambda^2}{2}]dt + \sigma_1S(S - \frac{\Lambda}{\mu} + W)dB_1 + \sigma_2W(S - \frac{\Lambda}{\mu} + W)dB_2.$$

因此考虑函数 $V_3 = V_{11} + V_{12} + V_{13} + V_{14} + V_{15}$, 计算得:

$$\begin{aligned} dV_3 = dV_{11} + dV_{12} + dV_{13} + dV_{14} + dV_{15} \leq & [2\mu(S - \frac{\Lambda}{\mu})^2 - (\mu + \frac{\rho}{2} - \frac{2 + \sigma_2^2}{2} - \frac{\beta_1\Lambda}{2\mu})W^2 - (\mu + \frac{\sigma}{2} - \frac{\rho + \sigma_3^2}{2} - \frac{\beta_2\Lambda}{2\mu})E^2 - (\mu + \alpha + \frac{\gamma}{2} - \frac{\sigma + \sigma_4^2}{2} - \frac{\beta_3\Lambda}{2\mu})I^2 - (\mu - \frac{\gamma + \sigma_5^2}{2})A^2]dt + \sigma_4I^2dB_4 + \\ & \sigma_1S(S - \frac{\Lambda}{\mu})dB_1 + \sigma_5A^2dB_5 + \sigma_1S(S - \frac{\Lambda}{\mu} + W)dB_1 + \sigma_2W(S - \frac{\Lambda}{\mu} + W)dB_2 + \\ & \sigma_3E^2dB_3 + \frac{\Lambda^2\sigma_1^2}{\mu^2} + \frac{\Lambda^2(\mu + \rho)^2}{2\mu^2} + \frac{\Lambda^2}{2} + \frac{\Lambda^3(\beta_1 + \beta_2 + \beta_3)}{2\mu^3}. \end{aligned} \quad (4)$$

对式(4)两端分别从 0 到 t 积分并取期望得

$$\begin{aligned} EV_3(t) - EV_3(0) \leq E \int_0^t & [-2\mu(S(\xi) - \frac{\Lambda}{\mu})^2 - (\mu + \frac{\rho}{2} - \frac{2 + \sigma_2^2}{2} - \frac{\beta_1\Lambda}{2\mu})W^2(\xi) - (\mu + \frac{\sigma}{2} - \frac{\rho + \sigma_3^2}{2} - \frac{\beta_2\Lambda}{2\mu})E^2(\xi) - \\ & (\mu + \alpha + \frac{\gamma}{2} - \frac{\sigma + \sigma_4^2}{2} - \frac{\beta_3\Lambda}{2\mu})I^2(\xi) - (\mu - \frac{\gamma + \sigma_5^2}{2})A^2(\xi)]d\xi + \sigma_3E \int_0^t E(\xi)^2dB_3 + \sigma_4E \int_0^t I(\xi)^2dB_4 + \\ & \sigma_1E \int_0^t S(\xi)(S(\xi) - \frac{\Lambda}{\mu})dB_1 + \sigma_1E \int_0^t S(\xi)(S(\xi) - \frac{\Lambda}{\mu} + W(\xi))dB_1 + \sigma_5E \int_0^t A(\xi)^2dB_5 + \\ & \sigma_2E \int_0^t W(\xi)(S(\xi) - \frac{\Lambda}{\mu} + W(\xi))dB_2 + E(\frac{\Lambda^2\mu(2\sigma_1^2 + (\mu + \rho)^2 + \mu^2) + \Lambda^3(\beta_1 + \beta_2 + \beta_3)}{2\mu^3}). \end{aligned}$$

由于 $\lim_{t \rightarrow \infty} \frac{\langle \int_0^t dB_i(t), \int_0^t dB_i(t) \rangle_t}{t} \leq 1 < \infty$, 因此 $\lim_{t \rightarrow \infty} \frac{\int_0^t dB_i(t)}{t} = 0 (i = 1, 2, \dots, 5)$ a.s., 所以

$$\limsup_{t \rightarrow \infty} \frac{1}{t} E \int_0^t [(S(\xi) - \frac{\Lambda}{\mu})^2 + W^2(\xi) + E^2(\xi) + I^2(\xi) + A^2(\xi)] d\xi \leq$$

$$\frac{\Lambda^2 \mu (2\sigma_1^2 + (\mu + \rho)^2 + \mu^2) + \Lambda^3 (\beta_1 + \beta_2 + \beta_3)}{2n_1 \mu^3}.$$

注 定理 4 表明,在一定条件下,随机模型(2)的解在确定性模型(1)的无病平衡点附近扰动,扰动程度与 $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5$ 相关.

4 随机模型(2)的解在地方病平衡点附近的渐近行为

定理 5 当 $R_0 > 1$ 时,对任意初值 $(S(0), W(0), E(0), I(0), A(0)) \in \mathbf{R}_+^5$,随机模型(2)的解 $(S(t), W(t), E(t), I(t), A(t))$ 有如下性质:

$$\limsup_{t \rightarrow \infty} \frac{1}{t} E \int_0^t [(S(\xi) - S^*)^2 + (W(\xi) - W^*)^2 + (E(\xi) - E^*)^2 + (I(\xi) - I^*)^2 + (A(\xi) - A^*)^2] d\xi \leq \frac{\delta}{n_2}.$$

其中: $n_2 = \min\{C_5 \frac{\mu}{2}, C_6 \frac{(\mu + \rho)}{4}, C_7 \frac{(\mu + \sigma)}{4}, C_8 \frac{(\mu + \alpha + \gamma)}{4}, C_9 \frac{\mu}{2}\}$, $\delta = \sigma_1^2 (\frac{C_1}{2} S^* + \frac{(C_5 + C_6) \Lambda^2}{2\mu^2}) + \sigma_2^2 (\frac{C_2}{2} W^* + \frac{C_6 \Lambda^2}{2\mu^2}) + \sigma_3^2 (\frac{(C_3 + C_4)}{2(\mu + \sigma)} + \frac{C_7 \Lambda^2}{2\mu^2}) + \sigma_4^2 (\frac{C_4}{2(\mu + \alpha + \gamma)} + \frac{C_8 \Lambda^2}{2\mu^2}) + \sigma_5^2 \frac{C_9 \Lambda^2}{2\mu^2}$,

$$C_1 = \frac{\beta_1 W^* + \beta_2 E^* + \beta_3 I^*}{\beta_1 W^* + \beta_2 E^* + \beta_3 I^* + \mu}, C_2 = 1, C_3 = \beta_2 S^* E^*, C_4 = \beta_3 S^* I^*, C_5 = \frac{\Lambda}{\mu}, C_6 = \frac{\mu(\mu + \rho)}{\mu^2 + (\mu + \rho)^2} C_5,$$

$$C_7 = \frac{(\mu + \sigma)(\mu + \rho)}{2\rho^2} C_6, C_8 = \frac{(\mu + \sigma)(\mu + \alpha + \gamma)}{2\sigma^2} C_7, C_9 = \frac{\mu(\mu + \alpha + \gamma)}{2\gamma^2} C_8.$$

证明 定义函数 $V_{21} = S - S^* - S^* \ln \frac{S}{S^*}$, $V_{22} = W - W^* - W^* \ln \frac{W}{W^*}$, $V_{23} = \frac{1}{\rho W^*} (E - E^* - E^* \ln \frac{E}{E^*})$, $V_{24} = \frac{1}{\rho W^*} (E - E^* - E^* \ln \frac{E}{E^*}) + \frac{1}{\sigma E^*} (I - I^* - I^* \ln \frac{I}{I^*})$, $V_{25} = \frac{1}{2} (S - S^*)^2$, $V_{26} = \frac{1}{2} (S - S^* + W - W^*)^2$, $V_{27} = \frac{1}{2} (E - E^*)^2$, $V_{28} = \frac{1}{2} (I - I^*)^2$, $V_{29} = \frac{1}{2} (A - A^*)^2$. 由 Itô 公式得:

$$dV_{21} = -[(\beta_1 W^* + \beta_2 E^* + \beta_3 I^* + \mu) \frac{(S - S^*)^2}{S} - \beta_1 (S - S^*) (W - W^*) - \beta_2 (S - S^*) (E - E^*) - \beta_3 (S - S^*) (I - I^*)] dt + \sigma_1 (S - S^*) dB_1,$$

$$dV_{22} \leq [\beta_1 (S - S^*) (W - W^*) + \beta_2 (S - S^*) (E - E^*) + \beta_3 (S - S^*) (I - I^*) + \beta_2 S^* E^* \frac{(S - S^*)^2}{S} + \beta_3 S^* I^* \frac{(I - I^*)^2}{S} + \beta_2 S^* E^* (\ln \frac{W}{W^*} - \frac{W}{W^*} - \ln \frac{E}{E^*} + \frac{E}{E^*}) + \beta_3 S^* I^* (\ln \frac{W}{W^*} - \frac{W}{W^*} - \ln \frac{I}{I^*} + \frac{I}{I^*}) + \frac{1}{2} \sigma_2^2 W^*] dt + \sigma_2 (W - W^*) dB_2,$$

$$dV_{23} \leq [\frac{W}{W^*} - \frac{E}{E^*} - \ln \frac{W}{W^*} + \ln \frac{E}{E^*} + \frac{E^*}{2\rho W^*} \sigma_3^2] dt + \frac{\sigma_3}{\rho W^*} (E - E^*) dB_3,$$

$$dV_{24} \leq [\frac{W}{W^*} - \frac{I}{I^*} - \ln \frac{I}{I^*} + \frac{E^*}{2\rho W^*} \sigma_3^2 + \frac{I^*}{2\sigma E^*} \sigma_4^2] dt + \frac{\sigma_3}{\rho W^*} (E - E^*) dB_3 + \frac{\sigma_4}{\sigma E^*} (I - I^*) dB_4,$$

$$dV_{25} \leq [-\mu (S - S^*)^2 - \beta_1 (S - S^*) (W - W^*) - \beta_2 (S - S^*) (E - E^*) - \beta_3 (S - S^*) (I - I^*) + \frac{1}{2} \sigma_1^2 S^2] dt + \sigma_1 S (S - S^*) dB_1,$$

$$dV_{26} \leq [-\mu (\frac{2\mu + \rho}{2(\mu + \rho)}) (S - S^*)^2 - \frac{(\mu + \rho)}{2} (W - W^*)^2 + \frac{1}{2} \sigma_1^2 S^2 + \frac{1}{2} \sigma_2^2 W^2] dt + \sigma_1 S (S - S^* + W - W^*) dB_1 + \sigma_2 W (S - S^* + W - W^*) dB_2,$$

$$dV_{27} \leq [\frac{\rho^2}{2(\mu + \sigma)} (W - W^*)^2 - \frac{(\mu + \sigma)}{2} (E - E^*)^2 + \frac{1}{2} \sigma_3^2 E^2] dt + \sigma_3 E (E - E^*) dB_3,$$

$$dV_{28} \leq \left[\frac{\sigma^2}{2(\mu + \alpha + \gamma)} (E - E^*) - \frac{(\mu + \alpha + \gamma)}{2} (I - I^*)^2 + \frac{1}{2} \sigma_4^2 I^2 \right] dt + \sigma_4 I (I - I^*) dB_4,$$

$$dV_{29} \leq \left[\frac{\gamma}{2\mu} (I - I^*) - \frac{\mu}{2} (A - A^*) + \frac{1}{2} \sigma_5^2 A^2 \right] dt + \sigma_5 A (A - A^*) dB_5.$$

因此,考虑函数 $V_4(S, W, E, I, A) = C_1 V_{21} + C_2 V_{22} + C_3 V_{23} + C_4 V_{24} + C_5 V_{25} + C_6 V_{26} + C_7 V_{27} + C_8 V_{28} + C_9 V_{29}$, 由 Itô 公式可得:

$$\begin{aligned} dV_4 \leq & -C_5 \frac{\mu}{2} (S - S^*) - C_6 \frac{(\mu + \rho)}{2} (W - W^*) - C_7 \frac{(\mu + \sigma)}{2} (E - E^*)^2 - C_8 \frac{(\mu + \alpha + \gamma)}{2} (I - I^*)^2 - \\ & C_9 \frac{\mu}{2} (A - A^*)^2 + \delta + C_1 \sigma_1 S (S - S^*) dB_1 + C_2 \sigma_2 (W - W^*) dB_2 + \frac{\sigma_3}{\rho W^*} (C_3 + C_4) (E - E^*) dB_3 + \\ & C_4 \frac{\sigma_4}{\sigma E^*} (I - I^*) dB_4 + C_5 (S - S^*) \sigma_1 S dB_1 + C_6 (\sigma_1 S (S - S^* + W - W^*) dB_1 + \sigma_2 W (S - S^* + \\ & W - W^*) dB_2) + C_7 \sigma_3 E (E - E^*) dB_3 + C_8 \sigma_4 I (I - I^*) dB_4 + C_9 \sigma_5 A (A - A^*) dB_5. \end{aligned}$$

对上式两端分别从 0 到 t 积分并取期望得: $EV_4(S(t), W(t), E(t), I(t), A(t)) - EV_4(S(0), W(0), E(0), I(0), A(0)) \leq E \int_0^t \left[-\frac{C_5 \mu}{2} (S(\xi) - S^*)^2 - \frac{C_6 (\mu + \rho)}{2} (W(\xi) - W^*)^2 - \frac{C_7 (\mu + \sigma)}{2} (E(\xi) - E^*)^2 - \frac{C_8 (\mu + \alpha + \gamma)}{2} (I(\xi) - I^*)^2 - \frac{C_9 \mu}{2} (A(\xi) - A^*)^2 \right] d\xi + \delta t$. 所以 $\limsup_{t \rightarrow \infty} \frac{1}{t} E \int_0^t [(S(\xi) - S^*)^2 + (W(\xi) - W^*)^2 + (E(\xi) - E^*)^2 + (I(\xi) - I^*)^2 + (A(\xi) - A^*)^2] d\xi \leq \frac{\delta}{n_2}$.

注 定理 5 表明,随机模型(2)的解在确定性模型(1)的地方病平衡点附近扰动,且扰动的强度与 $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5$ 成正比.

5 平均持续性

定理 6 若 $R_0 > 1$, 且 $\min\left\{\frac{C_5 \mu}{2} (S^*)^2, \frac{C_6 (\mu + \rho)}{2} (W^*)^2, \frac{C_7 (\mu + \sigma)}{2} (E^*)^2, \frac{C_8 (\mu + \alpha + \gamma)}{2} (I^*)^2, \frac{C_9 \mu}{2} (A^*)^2\right\} \geq \delta$ 则随机模型(2)的解平均持续, 即 $\liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t S(\xi) d\xi > 0, \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t W(\xi) d\xi > 0, \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t E(\xi) d\xi > 0, \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t I(\xi) d\xi > 0, \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t A(\xi) d\xi > 0, a.s.$

证明 令 $m = \min\left\{\frac{C_5 \mu}{2}, \frac{C_6 (\mu + \rho)}{2}, \frac{C_7 (\mu + \sigma)}{2}, \frac{C_8 (\mu + \alpha + \gamma)}{2}, \frac{C_9 \mu}{2}\right\}$, 因为 $R_0 > 1$, 由定理 5 得:

$$\liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t (S(\xi) - S^*)^2 d\xi \leq \frac{\delta}{m}, \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t (W(\xi) - W^*)^2 d\xi \leq \frac{\delta}{m}, \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t (E(\xi) - E^*)^2 d\xi \leq \frac{\delta}{m},$$

$$\liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t (I(\xi) - I^*)^2 d\xi \leq \frac{\delta}{m}, \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t (A(\xi) - A^*)^2 d\xi \leq \frac{\delta}{m}.$$

因为 $2S^* (S - S^*) \leq (S^*)^2 + (S - S^*)^2$, 即 $S \geq \frac{S^*}{2} - \frac{(S - S^*)^2}{2S^*}$, 所以: $\liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t S(\xi) d\xi \geq \frac{S^*}{2} -$

$\limsup_{t \rightarrow \infty} \frac{1}{t} \int_0^t \frac{(S(\xi) - S^*)^2}{2S^*} d\xi \geq \frac{S^*}{2} - \frac{\delta}{2S^* m} > 0$, 同理可得:

$$\liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t W(\xi) d\xi > 0, \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t E(\xi) d\xi > 0, \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t I(\xi) d\xi > 0, \liminf_{t \rightarrow \infty} \frac{1}{t} \int_0^t A(\xi) d\xi > 0.$$

6 随机灭绝性

本节将讨论疾病的灭绝性.

定理 7 对随机模型(2)的任意具有正初值的解 $(S(t), W(t), E(t), I(t), A(t)) \in \mathbf{R}_+^5$, 若 $\mu\omega_1 > \beta_1\Lambda$, $\omega_2[\mu\omega_1 - \beta_1\Lambda] > \beta_2\Lambda\rho$, 则有:(1) 当 $R_0 < 1$ 时, $(W(t), E(t), I(t))$ 指数收敛于 $(0, 0, 0)$ a.s.(2) 当 $R_0 > 1$ 且 $\frac{1}{3\theta_2^2}(\frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\theta_1^2\sigma_3^2 \wedge \frac{1}{2}\theta_2^2\sigma_4^2) > M$ 时, $(W(t), E(t), I(t))$ 指数收敛于 $(0, 0, 0)$ a.s., 即艾滋病灭绝. 其中:

$$M \geq \min\{\omega_1(R_0 - 1), \frac{\mu\omega_1\omega_2(R_0 - 1)}{\mu\omega_1 - \beta_1\Lambda}, \frac{\mu\omega_1\omega_2\omega_3(R_0 - 1)}{\omega_2[\mu\omega_1 - \beta_1\Lambda] - \beta_2\Lambda\rho}\},$$

$$\frac{\mu\omega_1 - \beta_1\Lambda}{\mu\rho} < \theta_1 < \frac{\beta_2\Lambda\omega_3 + \beta_3\Lambda\sigma}{\mu\omega_2\omega_3}, \frac{\omega_2[\mu\omega_1 - \beta_1\Lambda] - \beta_2\Lambda\rho}{\mu\rho\sigma} < \theta_2 < \frac{\beta_3\Lambda}{\mu\omega_3}.$$

证明 定义函数 $Q(t) = \ln(W(t) + \theta_1 E(t) + \theta_2 I(t))$, 由 Itô 公式得:

$$d\ln Q(t) = LVdt + \frac{\sigma_2 W dB_2 + \theta_1 E dB_3 + \theta_2 I dB_4}{W(t) + \theta_1 E(t) + \theta_2 I(t)}. \text{其中:}$$

$$LV = \frac{1}{W(t) + \theta_1 E(t) + \theta_2 I(t)} [\beta_1 SW + \beta_2 SE + \beta_3 SI - \omega_1 W + \theta_1(\rho W - \omega_2 E) + \theta_2(\sigma E - \omega_3 I)] - \frac{\sigma_2^2 W^2 + \theta_1^2 \sigma_3^2 E^2 + \theta_2^2 \sigma_4^2 I^2}{2(W(t) + \theta_1 E(t) + \theta_2 I(t))^2} \leq \frac{1}{W(t) + \theta_1 E(t) + \theta_2 I(t)} [(\beta_1 \frac{\Lambda}{\mu} - \omega_1 + \theta_1 \rho)W + (\beta_2 \frac{\Lambda}{\mu} - \theta_1 \omega_2 + \theta_2 \sigma)E + (\beta_3 \frac{\Lambda}{\mu} - \theta_2 \omega_3)I] - \frac{1}{3\theta_2^2}(\frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\theta_1^2\sigma_3^2 \wedge \frac{1}{2}\theta_2^2\sigma_4^2).$$

下面将 $\beta_1 \frac{\Lambda}{\mu} - \omega_1 + \theta_1 \rho, \beta_2 \frac{\Lambda}{\mu} - \theta_1 \omega_2 + \theta_2 \sigma, \beta_3 \frac{\Lambda}{\mu} - \theta_2 \omega_3$ 分为大于 0 与小于 0 进行讨论. 当 $R_0 < 1$ 时, 取

$$\frac{\beta_2\Lambda\omega_3 + \beta_3\Lambda\sigma}{\mu\omega_2\omega_3} < \theta_1 < \frac{\mu\omega_1 - \beta_1\Lambda}{\mu\rho}, \frac{\beta_3\Lambda}{\mu\omega_3} < \theta_2 < \frac{\omega_2[\mu\omega_1 - \beta_1\Lambda] - \beta_2\Lambda\rho}{\mu\rho\sigma}.$$

所以存在正数 m_1 使得 $\beta_1 \frac{\Lambda}{\mu} - \omega_1 + \theta_1 \rho < -m_1, \beta_2 \frac{\Lambda}{\mu} - \theta_1 \omega_2 + \theta_2 \sigma < -m_1 \theta_1, \beta_3 \frac{\Lambda}{\mu} - \theta_2 \omega_3 < -m_1 \theta_2$ 同时

成立. 则可取 $m_1 = \min\{\omega_1(1 - R_0), \frac{\mu\omega_1\omega_2(1 - R_0)}{\mu\omega_1 - \beta_1\Lambda}, \frac{\mu\omega_1\omega_2\omega_3(1 - R_0)}{\omega_2[\mu\omega_1 - \beta_1\Lambda] - \beta_2\Lambda\rho}\}$. 因此

$$d\ln Q(t) \leq [-m_1 - \frac{1}{3\theta_2^2}(\frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\theta_1^2\sigma_3^2 \wedge \frac{1}{2}\theta_2^2\sigma_4^2)]dt + \frac{\sigma_2 W dB_2 + \theta_1 E dB_3 + \theta_2 I dB_4}{W(t) + \theta_1 E(t) + \theta_2 I(t)}.$$

对上式两边从 0 到 t 积分得:

$$\ln Q(t) - \ln Q(0) \leq (-m_1 - \frac{1}{3\theta_2^2}(\frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\theta_1^2\sigma_3^2 \wedge \frac{1}{2}\theta_2^2\sigma_4^2))t + \int_0^t \frac{\sigma_2 W dB_2 + \theta_1 E dB_3 + \theta_2 I dB_4}{W(t) + \theta_1 E(t) + \theta_2 I(t)}.$$

所以 $\lim_{t \rightarrow \infty} W(t) = 0, \lim_{t \rightarrow \infty} E(t) = 0, \lim_{t \rightarrow \infty} I(t) = 0$. 同理, 当 $R_0 > 1$ 时, 取 $\frac{\mu\omega_1 - \beta_1\Lambda}{\mu\rho} < \theta_1 < \frac{\beta_2\Lambda\omega_3 + \beta_3\Lambda\sigma}{\mu\omega_2\omega_3}$,

$\frac{\omega_2[\mu\omega_1 - \beta_1\Lambda] - \beta_2\Lambda\rho}{\mu\rho\sigma} < \theta_2 < \frac{\beta_3\Lambda}{\mu\omega_3}$. 所以存在正数 m_2 使得 $\beta_1 \frac{\Lambda}{\mu} - \omega_1 + \theta_1 \rho > m_2, \beta_2 \frac{\Lambda}{\mu} - \theta_1 \omega_2 + \theta_2 \sigma >$

$m_2 \theta_1, \beta_3 \frac{\Lambda}{\mu} - \theta_2 \omega_3 > m_2 \theta_2$ 同时成立. 则可取 $m_2 = \min\{\omega_1(R_0 - 1), \frac{\mu\omega_1\omega_2(R_0 - 1)}{\mu\omega_1 - \beta_1\Lambda},$

$\frac{\mu\omega_1\omega_2\omega_3(R_0 - 1)}{\omega_2[\mu\omega_1 - \beta_1\Lambda] - \beta_2\Lambda\rho}\}$. 因此有

$$d\ln Q(t) \leq [M - \frac{1}{3\theta_2^2}(\frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\theta_1^2\sigma_3^2 \wedge \frac{1}{2}\theta_2^2\sigma_4^2)]dt + \frac{\sigma_2 W dB_2 + \theta_1 E dB_3 + \theta_2 I dB_4}{W(t) + \theta_1 E(t) + \theta_2 I(t)}, M \geq m_2.$$

对上式两边分别从 0 到 t 积分得:

$$\ln Q(t) - \ln Q(0) \leq (M - \frac{1}{3\theta_2^2}(\frac{1}{2}\sigma_2^2 \wedge \frac{1}{2}\theta_1^2\sigma_3^2 \wedge \frac{1}{2}\theta_2^2\sigma_4^2))t + \int_0^t \frac{\sigma_2 W dB_2 + \theta_1 E dB_3 + \theta_2 I dB_4}{W(t) + \theta_1 E(t) + \theta_2 I(t)}.$$

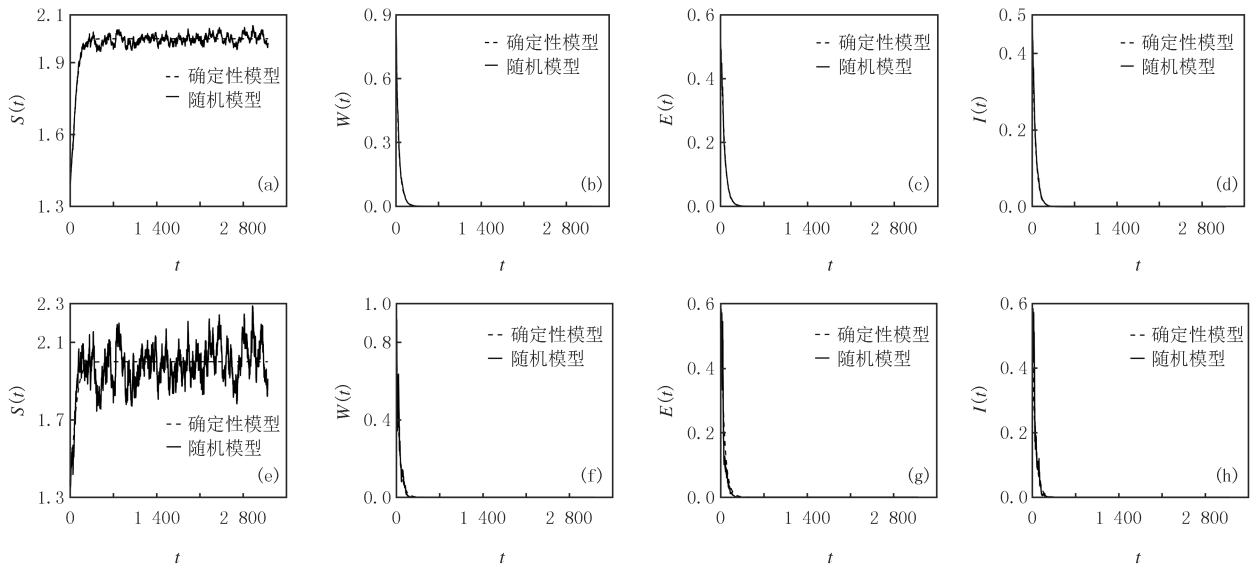
所以 $\lim_{t \rightarrow \infty} W(t) = 0, \lim_{t \rightarrow \infty} E(t) = 0, \lim_{t \rightarrow \infty} I(t) = 0$.

注 定理 7 表明,当 $R_0 > 1$ 时,确定性模型(1)的艾滋病还在流行,但随机模型(2)的艾滋病已经灭绝,说明强噪声可以使疾病灭绝.

7 数值模拟

利用 MATLAB 模拟确定性模型与随机模型(2)的解曲线,比较二者解的渐近行为之间的差异.取初值 $(S(0), W(0), E(0), I(0), A(0)) = (1.4, 0.9, 0.5, 0.5, 0.5)$.

(1)分别取参数 $\Lambda = 1, \mu = 0.5, \beta_1 = 0.05, \beta_2 = 0.1, \beta_3 = 0.2, \sigma = 0.3, \alpha = 0.1, \gamma = 0.2, \rho = 0.3, \sigma_1 = 0.01, \sigma_2 = 0.1, \sigma_3 = 0.1, \sigma_4 = 0.1, \sigma_5 = 0.1$,此时 $R_0 < 1$,且满足定理 4 的条件,模拟结果如图 1(a-d)所示.其他参数不变,取 $\sigma_1 = 0.1, \sigma_2 = 0.5, \sigma_3 = 5.0, \sigma_4 = 0.5, \sigma_5 = 0.5$,满足定理 4 的条件,模拟结果如图 1(e-h)所示.从图 1 可看到,随机模型(2)与确定性模型(1)均收敛于 P^0 ,且震荡幅度与 $\sigma_i (i = 1, 2, \dots, 5)$ 相关.



t :一个/组变量,在一系列时刻进行观测得到的一系列离散数字组合成的时间序列,下同.

图1 $R_0 < 1$ 时,随机模型(2)与确定性模型(1)解的渐近行为

Fig.1 The asymptotic behavior of solutions of stochastic model(2) and deterministic model(1) when $R_0 < 1$

(2)分别取参数 $\Lambda = 0.4, \mu = 0.1, \beta_1 = 0.1, \beta_2 = 0.3, \beta_3 = 0.4, \sigma = 0.1, \alpha = 0.05, \gamma = 0.2, \rho = 0.25, \sigma_1 = 0.005, \sigma_2 = 0.004, \sigma_3 = 0.005, \sigma_4 = 0.004, \sigma_5 = 0.003$,此时 $R_0 > 1$,模拟结果如图 2(a-d)所示.其他参数不变,取 $\sigma_1 = 0.05, \sigma_2 = 0.04, \sigma_3 = 0.05, \sigma_4 = 0.04, \sigma_5 = 0.03$,模拟结果如图 2(e-h)所示.从图 2 可观察到随机模型(2)与确定性模型(1)均收敛于 P^* ,且随机模型(2)的解围绕 P^* 做随机震荡.震荡幅度与 $\sigma_i (i = 1, 2, \dots, 5)$ 成正比.

(3)分别取参数 $\Lambda = 1, \mu = 0.1, \beta_1 = 0.01, \beta_2 = 0.02, \beta_3 = 0.05, \sigma = 0.3, \alpha = 0.1, \gamma = 0.2, \rho = 0.4, \sigma_1 = 0.005, \sigma_2 = 1.5, \sigma_3 = 1.3, \sigma_4 = 1.5, \sigma_5 = 0.8$,此时 $R_0 > 1$.取 $M = m_2, \theta_1 = 1.2, \theta_2 = 1.0$,满足定理 7 的条件,得到图 3.从图 3 中可以看出,此时虽确定性模型的解仍在地方病平衡点处稳定,但随机模型(2)中的艾滋病已经灭绝,由此可知,当随机干扰强度足够大时,可以导致 $W(t), E(t), I(t)$ 灭绝.

8 结论

本文首先研究了具有随机效应的 SWEIA 艾滋病毒传播模型,得到了确定性模型(1)平衡点的全局渐近稳定性与随机模型(2)正解的全局存在唯一性与有界性.其次,讨论了当 $R_0 < 1$ 时,随机模型(2)的解在相应确定性模型的无病平衡点附近扰动,且扰动程度与 $\sigma_i (i = 1, 2, \dots, 5)$ 相关;当 $R_0 > 1$ 时,随机模型(2)的解在相应确定性模型的地方病平衡点附近扰动,且扰动程度与 $\sigma_i (i = 1, 2, \dots, 5)$ 成正比.之后,分析了艾滋病传播的灭绝趋势,进而可以控制随机模型(2)中相应参数的大小,达到控制艾滋病传播的效果.最后,通过数值

模拟验证了理论结果.

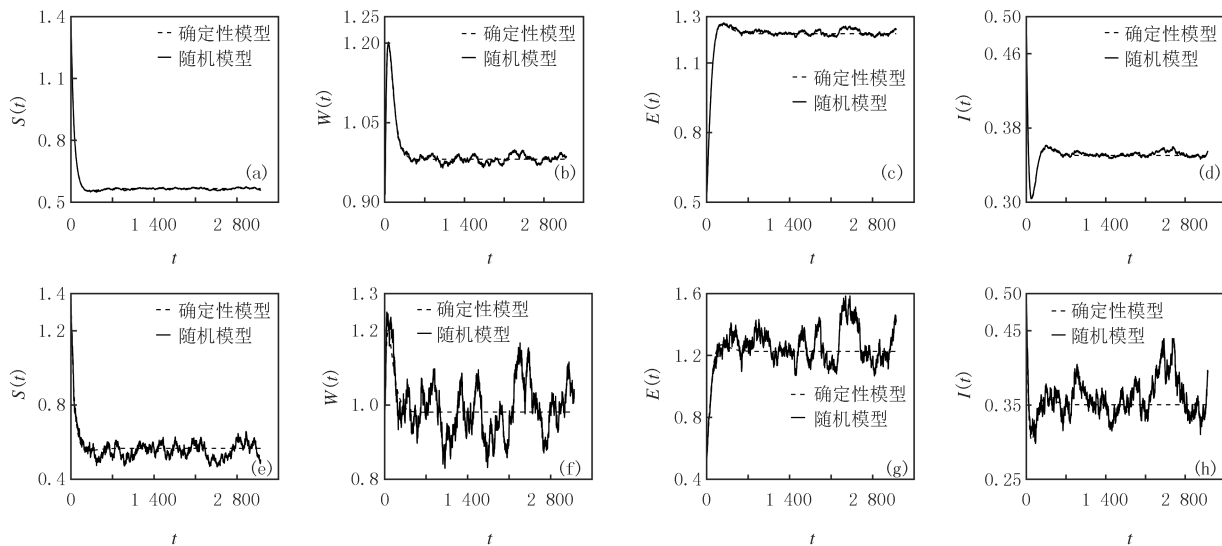


图2 $R_0 > 1$ 时, 随机模型(2)与确定性模型(1)的渐近行为

Fig.2 The asymptotic behavior of solutions of stochastic model(2) and deterministic model(1) when $R_0 > 1$

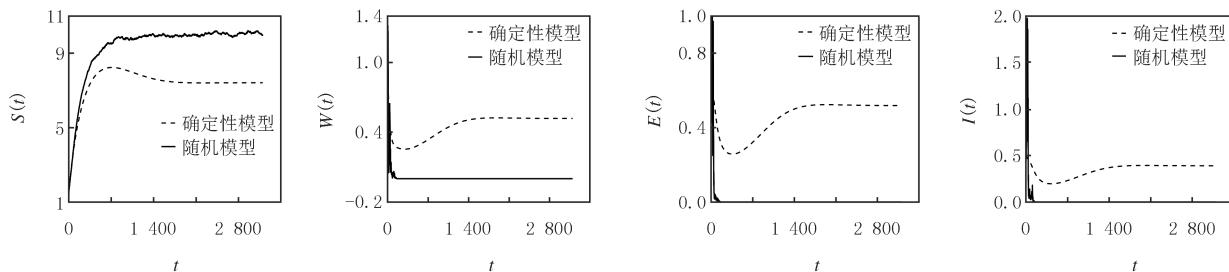


图3 强噪声时, 随机模型(2)与确定性模型(1)的解曲线

Fig.3 The solution curves of the stochastic model(2) and deterministic model(1) in strong noise

参 考 文 献

[1] 王楚雯,胡颖,侯颖.广西壮族自治区艾滋病模型及预测分析[J].检验检疫学刊,2020,30(2):6-9.
WANG C W, HU Y, HOU Y. Modeling and predictive analysis of AIDs in Guangxi Zhuang autonomous reigion[J]. Journal of Inspection and Quarantine, 2020, 30(2): 6-9.

[2] WU P, ZHAO H Y. Mathematical analysis of an age-structured HIV/AIDS epidemic model with HAART and spatial diffusion[J]. Non-linear Analysis: Real World Applications, 2021, 60: 103289.

[3] 张婷,张太雷.一类具有 VCT 意识的 HIV/AIDS 传染病模型[J].重庆理工大学学报(自然科学),2022,36(4):254-261.
ZHANG T, ZHANG T L. A class of HIV/AIDS epidemic model with VCT awareness[J]. Journal of Chongqing University of Technology (Natural Science), 2022, 36(4): 254-261.

[4] SHOFIANAH N, FITRI S, TRISILOWATI, et al. Optimal control of HIV/AIDS epidemic model with two latent stages, vertical transmission and treatment[J]. Journal of Physics: Conference Series, 2020, 1562(1): 012017.

[5] MARSUDI, TRISILOWATI, SURYANTO A, et al. Global stability and optimal control of an HIV/AIDS epidemic model with behavioral change and treatment[J]. Engineering Letters, 2021, 29(2): 575-591.

[6] BARDINA X, FERRANTE M, ROVIRA C. A stochastic epidemic model of COVID-19 disease[J]. AIMS Mathematics, 2020, 5(6): 7661-7677.

[7] RAZA A, RAFIQ M, BALEANU D, et al. Competitive numerical analysis for stochastic HIV/AIDS epidemic model in a two-sex population[J]. IET Systems Biology, 2019, 13(6): 305-315.

[8] 张启敏,曹博强,牟晓洁.具有信息干预的随机 SIRS 传染病模型正解的存在性与灭绝性[J].河南师范大学学报(自然科学版),2018, 46(4):1-7.
ZHANG Q M, CAO B Q, MU X J. The existence of positive solution and extinction of a stochastic SIRS epidemic model with information

- intervention[J].Journal of Henan Normal University(Natural Science Edition),2018,46(4):1-7.
- [9] RIFHAT R,TENG Z D,WANG C X.Extinction and persistence of a stochastic SIRV epidemic model with nonlinear incidence rate[J].Advances in Difference Equations,2021,2021(1):1-21.
- [10] ZHANG Y H,MA X S,DIN A.Stationary distribution and extinction of a stochastic SEIQ epidemic model with a general incidence function and temporary immunity[J].AIMS Mathematics,2021,6(11):12359-12378.
- [11] 郭晓霞,孙树林.混合随机 SIR 传染病模型的动力学分析[J].系统科学与数学,2022,42(4):992-1010.
GUO X X,SUN S L.Dynamic analysis of a hybrid stochastic SIR epidemic model[J].Journal of Systems Science and Mathematical Sciences,2022,42(4):992-1010.
- [12] 胡瑞,黄立冬,李荣庭,等.一类潜伏期与染病期均具有传染性的随机传染病模型[J].云南民族大学学报(自然科学版),2022,31(2):213-220.
HU R,HUANG L D,LI R T,et al.A stochastic epidemic model with infectivity in both incubation period and infection period[J].Journal of Yunnan Minzu University(Natural Sciences Edition),2022,31(2):213-220.
- [13] KHAN A,IKRAM R,DIN A,et al.Stochastic COVID-19 SEIQ epidemic model with time-delay[J].Results in Physics,2021,30:104775.
- [14] HOU T F,LAN G J,YUAN S L,et al.Threshold dynamics of a stochastic SIHR epidemic model of COVID-19 with general population-size dependent contact rate[J].Mathematical Biosciences and Engineering,2022,19(4):4217-4236.
- [15] 龚道远,李智明,张宇翔,等.SWEIA 艾滋病毒传染模型及应用[J].数学的实践与认识,2017,47(6):170-179.
GONG D Y,LI Z M,ZHANG Y X,et al.SWEIA HIV/AIDS epidemic model and application[J].Mathematics in Practice and Theory,2017,47(6):170-179.

Dynamic analysis of an SWEIA HIV infection model with stochastic effects

Ma Yiting, Zhang Tailei, Deng Jinchao

(School of Science, Chang'an University, Xi'an 710064, China)

Abstract: An SWEIA HIV epidemic model with stochastic effects is studied. Firstly, the global asymptotic stability of the equilibrium of the deterministic model is proved by constructing Lyapunov function, and the global existence, uniqueness, and boundedness of the positive solution of the stochastic model are proved by using stopping time theory. Secondly, the oscillation behavior of the solution of the stochastic model around the disease-free equilibrium and endemic equilibrium of the corresponding deterministic model is analyzed, and the sufficient conditions for the mean persistence and extinction of the solution of the stochastic model are obtained. Finally, the numerical simulation further shows the dynamic behavior of the model.

Keywords: stochastic model; Itô formula; oscillating behavior; persistence; extinction

[责任编辑 陈留院 赵晓华]