2-D 离散时滞系统的新时滞相关稳定性准则

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摘　要:研究了具有时变时滞的二维(two-dimensional,2-D)离散系统的时滞相关稳定性问题.所创建的 Lyapunov-Krasovskii 泛函(Lyapunov-Krasovskii functionals,LKFs)考虑在二次项和单项求和项中引入时滞相关矩阵,包含了更多的状态信息.同时在单项求和项中引入增广向量矩阵,并给出适用于 2-D 系统的多重辅助函数不等式和互凸组合不等式,用于处理 LKFs 差分,以便降低计算负担.然后,为具有时变时滞的 2-D 离散系统推导出保守性更小的稳定性准则.通过两个数值算例验证了所设计方法的有效性和优越性.

关键词:二维离散系统;时变时滞;Lyapunov-Krasovskii 泛函;多重辅助函数不等式;互凸组合不等式

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二维(2-D)系统通常被视为有两个独立的自变量.由于其在自动控制、迭代学习控制、多维数字滤波器等多个学科和工程领域发挥着重要作用,2-D 离散系统越来越受到关注[1-2].因此,许多学者对 2-D 离散系统进行了广泛研究,并取得了许多成果[3].

众所周知,在很多实际系统中,时滞都是不可避免的,例如通信系统,电力系统,网络传输系统[4-6].所以时滞是现实生活以及实际工程系统中有待解决的问题[7-11].时滞的存在,一方面使系统的动态性能变差,甚至造成系统的不稳定.另一方面,在一些控制系统中,人们可以利用时滞来提高控制效果,比如在重复控制系统中,需要利用时滞来达到预期目标[12-15].为了更好地利用时滞解决实际问题,避免其不良后果,有必要从理论角度更加深入地分析和理解时滞对动态系统的影响.

由于离散系统更适用于实际生活而逐渐受到更多的关注,离散系统的稳定性也因此成为一个热门话题.因此,针对 1-D 离散系统中 LKFs 的设计方面已经得到突破[16-23].文献[24]构建了一个新的 LKFs,其中包含两个时滞相关矩阵,一个具有单项求和项,另一个具有二次项,用来研究离散时滞系统的稳定性.但是,由于 2-D 系统的结构,即信息传播发生在两个独立的方向上,选取的 LKFs 通常导致系统的保守性较大[25].目前对于 2-D 离散系统的研究,文献[26]采用 LKFs 方法研究了 2-D 连续离散系统的有限区域耗散控制问题.文献[27]给出了 2-D 离散系统的稳定性准则,其中 LKFs 是使用 LMIs 的区间时变时滞结合时滞分区的方法所设计.文献[28]研究了基于时滞分区的 LKFs 来分析 2-D 离散系统的时滞相关稳定性问题.文献[29]构建的增广 LKFs 充分利用了时滞变化的信息.以上针对 1-D 离散系统研究的文献都在 LKFs 的构造中进行了创新,同时考虑到了更多的状态信息.然而,到目前为止,还没有关于在 2-D 离散系统中将时滞相关技术扩展到 LKFs 求和项的文章.基于上述分析,本文的主要创新点如下:

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1)建立具有时变时滞的 2-D 离散系统模型，将时滞相关矩阵 $\boldsymbol{P}_1(d_1(i))$，$\boldsymbol{P}_2(d_2(j))$，$\boldsymbol{Q}_1(d_1(i))$，$\boldsymbol{Q}_2(d_2(j))$ 和增广向量矩阵 $\boldsymbol{\xi}_k(i,j)(k=1,2,3,4)$ 分别添加到二次项和单项求和项中，同时所创建的 LKFs 还包含三重求和项，例如 $\bar{V}_5=\sum\limits_{s=-d_{1M}}^{-d_{1m}-1}\sum\limits_{u=s}^{-d_{1m}-1}\sum\limits_{\nu=u}^{-1}y^{\mathrm{T}}(i+\nu,j)\boldsymbol{R}_1y(i+\nu,j)$（与水平方向相关），这包含了有关时滞上界和下界的更多信息，从而得到保守性更小的稳定性准则和较大的时滞上界.

2)新的 2-D 加权求和不等式应用于 LKFs 前向差分中有限和项的处理，加权不等式的存在简化了计算过程，降低了系统稳定性准则的计算负担，同时促进了 2-D 系统理论的发展.然后根据 LMIs 推导出新的时滞相关稳定性准则.与参考文献中时滞相关的稳定性结果相比，本文推导的结果利用的决策变量的数目更少，并且适用于更广泛的时滞范围.

1　模型描述

考虑 2-D 离散时变时滞系统，如下：

$$\begin{aligned}x(i+1,j+1)=&\boldsymbol{A}_1x(i,j+1)+\boldsymbol{A}_2x(i+1,j)+\boldsymbol{A}_{1d}x(i-d_1(i),j+1)+\\&\boldsymbol{A}_{2d}x(i+1,j-d_2(j)),\end{aligned}\tag{1}$$

其中 $x(i,j)\in\boldsymbol{R}^n$ 是状态向量，$i,j\in\mathbf{N}$.$\boldsymbol{A}_1$，$\boldsymbol{A}_2$，$\boldsymbol{A}_{1d}$ 和 $\boldsymbol{A}_{2d}$ 是具有适当维度的常量矩阵.$d_1(i)$ 和 $d_2(j)$ 分别是水平和竖直方向上的时变时滞，分别满足：

$$\begin{gathered}0<d_{1m}\leq d_1(i)\leq d_{1M},0<d_{2m}\leq d_2(j)\leq d_{2M},\\\mu_{1m}\leq\Delta d_1(i)=d_1(i+1)-d_1(i)\leq\mu_{1M},\mu_{2m}\leq\Delta d_2(j)=d_2(j+1)-d_2(j)\leq\mu_{2M},\end{gathered}\tag{2}$$

其中 d_{1m}，d_{2m}，d_{1M} 和 d_{2M} 是常量正整数，表示时滞边界，μ_{1m}，μ_{2m}，μ_{1M} 和 μ_{2M} 是常量整数，表示时滞变化范围.边界条件假定为：

$$\begin{cases}x(i,j)=\varphi_{i,j},\forall\,0\leq i\leq r_1,j=-d_{2M},-d_{2M}+1,\cdots,0,\\x(i,j)=0,\forall\,i>r_1,j=-d_{2M},-d_{2M}+1,\cdots,0,\\x(i,j)=\psi_{i,j},\forall\,0\leq j\leq r_2,i=-d_{1M},-d_{1M}+1,\cdots0,\\x(i,j)=0,\forall\,j>r_2,i=-d_{1M},-d_{1M}+1,\cdots,0,\\\varphi_{0,0}=\psi_{0,0},\end{cases}\tag{3}$$

其中 r_1 和 r_2 是正整数.

鉴于上述形式，本文旨在找到新的稳定性准则保证系统(1)稳定.为了推导出本文结果，提供了适用于 2-D系统的多重辅助函数不等式[30]以及互凸组合不等式[31].

引理 1[30]　对于给定的正定 $n\times n$ 矩阵 $\boldsymbol{R}$，3 个给定的非负整数 a,b,k 满足 $a<b\leq k$，一个向量函数 $x(\cdot)\in\boldsymbol{R}^n$ 并且表示 $y(s,j)=x(s+1,j)-x(s,j)$，$y(i,s)=x(i+1,s)-x(i,s)$，有：

$$\begin{aligned}&1)\sum_{s=-b}^{-a-1}y^{\mathrm{T}}(s,1)\boldsymbol{R}y(s,1)\geq\frac{1}{b-a}(\bar{\Omega}^0_{a,b})^{\mathrm{T}}\boldsymbol{R}(\bar{\Omega}^0_{a,b})+\frac{3}{b-a}(\bar{\Omega}^1_{a,b})^{\mathrm{T}}\boldsymbol{R}(\bar{\Omega}^1_{a,b})+\frac{5}{b-a}(\bar{\Omega}^2_{a,b})^{\mathrm{T}}\boldsymbol{R}(\bar{\Omega}^2_{a,b}),\\&2)\sum_{s=-b}^{-a-1}\sum_{u=s}^{-a-1}y^{\mathrm{T}}(u,1)\boldsymbol{R}y(u,1)\geq2(\bar{\Omega}^3_{a,b})^{\mathrm{T}}\boldsymbol{R}(\bar{\Omega}^3_{a,b})+4(\bar{\Omega}^4_{a,b})^{\mathrm{T}}\boldsymbol{R}(\bar{\Omega}^4_{a,b}),\end{aligned}\tag{4}$$

其中

$$\bar{\Omega}^0_{a,b}=x(-a,1)-x(-b,1),\bar{\Omega}^1_{a,b}=x(-a,1)+x(-b,1)-\frac{2}{b-a+1}\sum_{u=-b}^{-a}x(u,1),$$

$$\bar{\Omega}^2_{a,b}=x(-a,1)-x(-b,1)+\frac{6}{b-a+1}\sum_{u=-b}^{-a}x(u,1)-\frac{12}{(b-a+1)(b-a+2)}\sum_{s=-b}^{-a}\sum_{u=s}^{-a}x(u,1),$$

$$\bar{\Omega}^3_{a,b}=x(-a,1)-\frac{1}{b-a+1}\sum_{u=-b}^{-a}x(u,1),$$

$$\bar{\Omega}^4_{a,b}=x(-a,1)+\frac{2}{b-a+1}\sum_{u=-b}^{-a}x(u,1)-\frac{6}{(b-a+1)(b-a+2)}\sum_{s=-b}^{-a}\sum_{u=s}^{-a}x(u,1).$$

引理 2[31]　对于任何向量 $\xi\in\boldsymbol{R}^{m_x}$，矩阵 $\boldsymbol{R}_1,\boldsymbol{R}_2\in\boldsymbol{S}^{n_x}_{+}$，$\boldsymbol{S}\in\boldsymbol{R}^{n_x\times n_x}$，$\boldsymbol{W}_1,\boldsymbol{W}_2\in\boldsymbol{R}^{n_x\times m_x}$ 和实标量 $\alpha>0$，

$\beta > 0$ 满足 $\alpha + \beta = 1$，满足以下不等式：

$$\frac{1}{\alpha}\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{W}_1^{\mathrm{T}}\boldsymbol{R}_1\boldsymbol{W}_1\boldsymbol{\xi} + \frac{1}{\beta}\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{W}_2^{\mathrm{T}}\boldsymbol{R}_2\boldsymbol{W}_2\boldsymbol{\xi} \geq \boldsymbol{\xi}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{W}_1\\ \boldsymbol{W}_2\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{R}_1 & \boldsymbol{S}\\ \boldsymbol{S}^{\mathrm{T}} & \boldsymbol{R}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{W}_1\\ \boldsymbol{W}_2\end{bmatrix}\boldsymbol{\xi}, \tag{5}$$

满足 $\begin{bmatrix}\boldsymbol{R}_1 & \boldsymbol{S}\\ \boldsymbol{S}^{\mathrm{T}} & \boldsymbol{R}_2\end{bmatrix} \geq 0$.

2 稳定性分析

本节给出了使用增广 LKFs 推导出的新稳定性准则.新型 LKFs 的形式如下：

$$V = \bar{V} + \hat{V} = \sum_{k=1}^{5}\bar{V}_k + \sum_{k=1}^{5}\hat{V}_k, \tag{6}$$

其中 $\bar{V}_1 = \boldsymbol{\xi}_1^{\mathrm{T}}\boldsymbol{P}_1(d_1(i))\boldsymbol{\xi}_1$，$\bar{V}_2 = \bar{d}_1\sum_{s=-d_{1M}}^{-d_{1m}+1}\boldsymbol{\xi}_3^{\mathrm{T}}(i+s,j)\boldsymbol{Q}_1(d_1(i))\boldsymbol{\xi}_3(i+s,j)$，$\bar{V}_3 = \bar{d}_1\sum_{s=-d_{1M}}^{-1}\sum_{u=s}^{-1}y^{\mathrm{T}}(i+u,j)\boldsymbol{Q}_3 y(i+u,j)$，$\bar{V}_4 = \sum_{s=-d_{1M}}^{-1}\sum_{u=s}^{-1}\sum_{\nu=u}^{-1}y^{\mathrm{T}}(i+\nu,j)\boldsymbol{S}_1 y(i+\nu,j)$，$\bar{V}_5 = \sum_{s=-d_{1M}}^{-d_{1m}-1}\sum_{u=s}^{-d_{1m}-1}\sum_{\nu=u}^{-1}y^{\mathrm{T}}(i+\nu,j)\boldsymbol{R}_1 y(i+\nu,j)$，$\hat{V}_1 = \boldsymbol{\xi}_2^{\mathrm{T}}\boldsymbol{P}_2(d_2(j))\boldsymbol{\xi}_2$，$\hat{V}_2 = \bar{d}_2\sum_{s=-d_{2M}}^{-d_{2m}+1}\boldsymbol{\xi}_4^{\mathrm{T}}(i,j+s)\boldsymbol{Q}_2(d_2(j))\boldsymbol{\xi}_4(i,j+s)$，$\hat{V}_3 = \bar{d}_2\sum_{s=-d_{2M}}^{-1}\sum_{u=s}^{-1}y^{\mathrm{T}}(i,j+u)\boldsymbol{Q}_4 y(i,j+u)$，$\hat{V}_4 = \sum_{s=-d_{2M}}^{-1}\sum_{u=s}^{-1}\sum_{\nu=u}^{-1}y^{\mathrm{T}}(i,j+\nu)\boldsymbol{S}_2 y(i,j+\nu)$，$\hat{V}_5 = \sum_{s=-d_{2M}}^{-d_{2m}-1}\sum_{u=s}^{-d_{2m}-1}\sum_{\nu=u}^{-1}y^{\mathrm{T}}(i,j+\nu)\boldsymbol{R}_2 y(i,j+\nu)$，以及

$$\boldsymbol{\xi}_1(i,j) = [x^{\mathrm{T}}(i,j) \quad x^{\mathrm{T}}(i-d_1(i),j) \quad \sum_{s=-d_{1m}}^{-1}]x^{\mathrm{T}}(i+s,j) \quad \sum_{s=-d_{1m}}^{-1}\sum_{u=s}^{-1}x^{\mathrm{T}}(i+u,j)]^{\mathrm{T}},$$

$$\boldsymbol{\xi}_2(i,j) = [x^{\mathrm{T}}(i,j) \quad x^{\mathrm{T}}(i,j-d_2(i)) \quad \sum_{s=-d_{2m}}^{-1}]x^{\mathrm{T}}(i,j+s) \quad \sum_{s=-d_{2m}}^{-1}\sum_{u=s}^{-1}x^{\mathrm{T}}(i,j+u)]^{\mathrm{T}},$$

$$\boldsymbol{\xi}_3(s,j) = [y(s,j) \quad x(s,j) \quad \sum_{u=s}^{-d_{1m}-1}y(u,j)]^{\mathrm{T}}, \boldsymbol{\xi}_4(i,s) = [y(i,s) \quad x(i,s) \quad \sum_{u=s}^{-d_{2m}-1}y(i,u)]^{\mathrm{T}}.$$

本文的创新之处是在创建 LKFs(6)时，常数矩阵 $\boldsymbol{P}_1$，$\boldsymbol{P}_2$ 和 $\boldsymbol{Q}_1$，$\boldsymbol{Q}_2$ 被时滞相关矩阵 $\boldsymbol{P}_1(d_1(i))$，$\boldsymbol{P}_2(d_2(j))$ 和 $\boldsymbol{Q}_1(d_1(i))$，$\boldsymbol{Q}_2(d_2(j))$ 所替代，这些矩阵包含更多的时变时滞信息.另一方面，增广向量矩阵 $\boldsymbol{\xi}_1(i,j)$，$\boldsymbol{\xi}_2(i,j)$，$\boldsymbol{\xi}_3(i,j)$，$\boldsymbol{\xi}_4(i,j)$ 被添加到单项求和项中.受文献[24]中结果的启发，二次项中的时滞相关矩阵 $\boldsymbol{P}_1(d_1(i))$，$\boldsymbol{P}_2(d_2(j))$，$\boldsymbol{Q}_1(d_1(i))$ 和 $\boldsymbol{Q}(d_2(j))$ 构造如下形式：

$$\begin{cases}\boldsymbol{P}_1(d_1(i)) = \begin{bmatrix}(d_{1M}-d_1(i))\boldsymbol{P}_{11}^1(i,0) + (d_1(i)-d_{1m})\boldsymbol{P}_{11}^2(i,0) & \boldsymbol{P}_{12}\\ * & \boldsymbol{P}_{22}\end{bmatrix},\\ \boldsymbol{P}_2(d_2(j)) = \begin{bmatrix}(d_{2M}-d_2(j))\boldsymbol{P}_{11}^1(0,j) + (d_2(j)-d_{2m})\boldsymbol{P}_{11}^2(0,j) & \boldsymbol{P}_{12}\\ * & \boldsymbol{P}_{22}\end{bmatrix},\end{cases} \tag{7}$$

$$\begin{cases}\boldsymbol{Q}_1(d_1(i)) = \begin{bmatrix}(d_1(i)-d_{1m})\boldsymbol{Q}_{11}^1(i,0) + (d_{1M}-d_1(i))\boldsymbol{Q}_{11}^2(i,0) + \boldsymbol{Q}_{10} & \boldsymbol{Q}_{12}\\ * & \boldsymbol{Q}_{22}\end{bmatrix},\\ \boldsymbol{Q}_2(d_2(j)) = \begin{bmatrix}(d_2(j)-d_{2m})\boldsymbol{Q}_{11}^1(0,j) + (d_{2M}-d_2(j))\boldsymbol{Q}_{11}^2(0,j) + \boldsymbol{Q}_{10} & \boldsymbol{Q}_{12}\\ * & \boldsymbol{Q}_{22}\end{bmatrix},\end{cases} \tag{8}$$

需要以下表示以便推导出结果：

$$\alpha_1 = \frac{d_1(i)-d_{1m}}{\bar{d}_1}, \beta_1 = \frac{d_{1M}-d_1(i)}{\bar{d}_1}, \mu_1 = \max\{|\mu_{1m}|, |\mu_{1M}|\}, \bar{\nu}_{a,b} = \frac{1}{b-a+1}\sum_{s=-b}^{-a}x(s,1),$$

$\alpha_2=\dfrac{d_2(j)-d_{2m}}{\bar{d}_2},\beta_2=\dfrac{d_{2M}-d_2(j)}{\bar{d}_2},\mu_2=\max\{|\mu_{2m}|,|\mu_{2M}|\},\hat{\nu}_{a,b}=\dfrac{1}{b-a+1}\sum_{s=-b}^{-a}x(1,s),$

$\bar{\sigma}_{a,b}=\dfrac{1}{(b-a+1)(b-a+2)}\sum_{s=-b}^{-a}\sum_{u=s}^{-a}x(s,1),\bar{d}_1=d_{1M}-d_{1m},\tilde{\boldsymbol{R}}_l=\mathrm{diag}\{\boldsymbol{R}_l,\boldsymbol{R}_l,\boldsymbol{R}_l\},l=1,2,$

$\hat{\sigma}_{a,b}=\dfrac{1}{(b-a+1)(b-a+2)}\sum_{s=-b}^{-a}\sum_{u=s}^{-a}x(1,s),\bar{d}_2=d_{2M}-d_{2m},\tilde{\boldsymbol{S}}_k=\mathrm{diag}\{\boldsymbol{S}_k,\boldsymbol{S}_k,\boldsymbol{S}_k\},k=1,2,$

$e_i=[0_{n\times(i-1)n}\quad I_n\quad 0_{n\times(32-i)n}],i=1,2,\cdots,32,e_s=A_1e_1^{\mathrm{T}}+A_2e_2^{\mathrm{T}}+A_{3d}e_3^{\mathrm{T}}+A_{4d}e_4^{\mathrm{T}},$

$\boldsymbol{\zeta}=\mathrm{col}\{x_{0,1},x_{1,0},x_{-d_1(i),1},x_{1,-d_2(j)},x_{-d_{1m},1},x_{1,-d_{2m}},x_{-d_{1M},1},x_{1,-d_{2M}},x_{-d_{1m}+1,1},x_{1,-d_{2m}+1},x_{-d_{1M}+1,1},x_{1,-d_{1M}+1},$
$\Delta x_{-d_1(i),1},\Delta x_{1,-d_2(j)},\Delta x_{0,1},\Delta x_{1,0},\bar{\nu}_{0,d_{1m}},\hat{\nu}_{0,d_{2m}},\bar{\sigma}_{0,d_{1m}},\hat{\sigma}_{0,d_{2m}},\bar{\nu}_{d_1(i),d_{1M}},\hat{\nu}_{d_2(j),d_{2M}},\bar{\nu}_{d_{1m},d_1(i)},\hat{\nu}_{d_{2m},d_2(j)},$
$\bar{\sigma}_{d_1(i),d_{1M}},\hat{\sigma}_{d_2(j),d_{2M}},\bar{\sigma}_{d_{1m},d_1(i)},\hat{\sigma}_{d_{2m},d_2(j)},\bar{\nu}_{0,d_{1M}},\hat{\nu}_{0,d_{2M}},\bar{\sigma}_{0,d_{1M}},\hat{\sigma}_{0,d_{2M}}\},$

$\Pi_0=[e_s^{\mathrm{T}}\quad e_3^{\mathrm{T}}+e_{13}^{\mathrm{T}}\quad (d_{1m}+1)e_{17}^{\mathrm{T}}-e_5^{\mathrm{T}}\quad (d_{1m}+1)(d_{1m}+2)e_{19}^{\mathrm{T}}-(d_{1m}+1)e_5^{\mathrm{T}}]^{\mathrm{T}},\rho_1=e_9-e_5,$

$\Pi_3=[e_s^{\mathrm{T}}\quad e_4^{\mathrm{T}}+e_{14}^{\mathrm{T}}\quad (d_{2m}+1)e_{18}^{\mathrm{T}}-e_6^{\mathrm{T}}\quad (d_{2m}+1)(d_{2m}+2)e_{20}^{\mathrm{T}}-(d_{1m}+1)e_6^{\mathrm{T}}]^{\mathrm{T}},\rho_2=e_{10}-e_6,$

$\Pi_1=[e_1^{\mathrm{T}}\quad e_3^{\mathrm{T}}\quad (d_{1m}+1)e_{17}^{\mathrm{T}}-e_1^{\mathrm{T}}\quad (d_{1m}+1)(d_{1m}+2)e_{19}^{\mathrm{T}}-(d_{1m}+1)e_1^{\mathrm{T}}]^{\mathrm{T}},\rho_{13}=e_s-e_1,$

$\Pi_4=[e_2^{\mathrm{T}}\quad e_4^{\mathrm{T}}\quad (d_{2m}+1)e_{18}^{\mathrm{T}}-e_2^{\mathrm{T}}\quad (d_{2m}+1)(d_{2m}+2)e_{20}^{\mathrm{T}}-(d_{2m}+1)e_2^{\mathrm{T}}]^{\mathrm{T}},\rho_{14}=e_s-e_2,$

$\Pi_2=[e_s^{\mathrm{T}}-e_1^{\mathrm{T}}\quad e_{13}^{\mathrm{T}}\quad e_1^{\mathrm{T}}-e_5^{\mathrm{T}}\quad (d_{1m}+1)e_1^{\mathrm{T}}-(d_{1m}+1)e_5^{\mathrm{T}}]^{\mathrm{T}},\Pi_7=[e_{11}^{\mathrm{T}}-e_7^{\mathrm{T}}\quad e_7^{\mathrm{T}}\quad e_5^{\mathrm{T}}-e_7^{\mathrm{T}}]^{\mathrm{T}},$

$\Pi_5=[e_s^{\mathrm{T}}-e_2^{\mathrm{T}}\quad e_{14}^{\mathrm{T}}\quad e_2^{\mathrm{T}}-e_6^{\mathrm{T}}\quad (d_{2m}+1)e_2^{\mathrm{T}}-(d_{2m}+1)e_6^{\mathrm{T}}]^{\mathrm{T}},\Pi_{11}=[e_{12}^{\mathrm{T}}-e_8^{\mathrm{T}}\quad e_8^{\mathrm{T}}\quad e_6^{\mathrm{T}}-e_8^{\mathrm{T}}]^{\mathrm{T}},$

$\Pi_9=[e_9^{\mathrm{T}}-e_{11}^{\mathrm{T}}\quad (d_{1M}-d_1(i)+1)e_{21}^{\mathrm{T}}+(d_1(i)-d_{1m}+1)e_{23}^{\mathrm{T}}-e_3^{\mathrm{T}}-e_7^{\mathrm{T}}\quad \bar{d}_1e_5^{\mathrm{T}}-(d_{1M}-$
$d_1(i)+1)e_{21}^{\mathrm{T}}-(d_1(i)-d_{1m}+1)e_{23}^{\mathrm{T}}+e_3^{\mathrm{T}}+e_7^{\mathrm{T}}]^{\mathrm{T}},\rho_{15}=[\rho_5^{\mathrm{T}}\quad \rho_3^{\mathrm{T}}]^{\mathrm{T}},$

$\Pi_{13}=[e_{10}^{\mathrm{T}}-e_{12}^{\mathrm{T}}\quad (d_{2M}-d_2(j)+1)e_{22}^{\mathrm{T}}+(d_2(j)-d_{1m}+1)e_{24}^{\mathrm{T}}-e_4^{\mathrm{T}}-e_8^{\mathrm{T}}\quad \bar{d}_2e_6^{\mathrm{T}}-(d_{2M}-$
$d_2(j)+1)e_{22}^{\mathrm{T}}-(d_2(j)-d_{1m}+1)e_{24}^{\mathrm{T}}+e_4^{\mathrm{T}}+e_8^{\mathrm{T}}]^{\mathrm{T}},\rho_{16}=[\rho_6^{\mathrm{T}}\quad \rho_4^{\mathrm{T}}]^{\mathrm{T}},$

$\rho_3=[e_3^{\mathrm{T}}-e_7^{\mathrm{T}}\quad \sqrt{3}(e_3^{\mathrm{T}}+e_7^{\mathrm{T}}-2e_{21}^{\mathrm{T}})\quad \sqrt{5}(e_3^{\mathrm{T}}-e_7^{\mathrm{T}}+6e_{21}^{\mathrm{T}}-12e_{25}^{\mathrm{T}})]^{\mathrm{T}},\Pi_6=[e_9^{\mathrm{T}}-e_5^{\mathrm{T}}\quad e_5^{\mathrm{T}}\quad 0]^{\mathrm{T}},$

$\rho_4=[e_4^{\mathrm{T}}-e_8^{\mathrm{T}}\quad \sqrt{3}(e_4^{\mathrm{T}}+e_8^{\mathrm{T}}-2e_{22}^{\mathrm{T}})\quad \sqrt{5}(e_4^{\mathrm{T}}-e_8^{\mathrm{T}}+6e_{22}^{\mathrm{T}}-12e_{26}^{\mathrm{T}})]^{\mathrm{T}},\Pi_{10}=[e_{10}^{\mathrm{T}}-e_6^{\mathrm{T}}\quad e_6^{\mathrm{T}}\quad 0]^{\mathrm{T}},$

$\rho_5=[e_5^{\mathrm{T}}-e_3^{\mathrm{T}}\quad \sqrt{3}(e_5^{\mathrm{T}}+e_3^{\mathrm{T}}-2e_{23}^{\mathrm{T}})\quad \sqrt{5}(e_5^{\mathrm{T}}-e_3^{\mathrm{T}}+6e_{23}^{\mathrm{T}}-12e_{27}^{\mathrm{T}})]^{\mathrm{T}},\Pi_8=[0\quad 0\quad e_9^{\mathrm{T}}-e_5^{\mathrm{T}}]^{\mathrm{T}},$

$\rho_6=[e_6^{\mathrm{T}}-e_4^{\mathrm{T}}\quad \sqrt{3}(e_6^{\mathrm{T}}+e_4^{\mathrm{T}}-2e_{24}^{\mathrm{T}})\quad \sqrt{5}(e_6^{\mathrm{T}}-e_4^{\mathrm{T}}+6e_{24}^{\mathrm{T}}-12e_{28}^{\mathrm{T}})]^{\mathrm{T}},\Pi_{12}=[0\quad 0\quad e_{10}^{\mathrm{T}}-e_6^{\mathrm{T}}]^{\mathrm{T}},$

$\rho_7=[\sqrt{2}(e_1^{\mathrm{T}}-e_{29}^{\mathrm{T}})\quad 2(e_1^{\mathrm{T}}+2e_{29}^{\mathrm{T}}-6e_{31}^{\mathrm{T}})]^{\mathrm{T}},\rho_9=[\sqrt{2}(e_3^{\mathrm{T}}-e_{21}^{\mathrm{T}})\quad 2(e_3^{\mathrm{T}}+2e_{21}^{\mathrm{T}}-6e_{25}^{\mathrm{T}})]^{\mathrm{T}},$

$\rho_8=[\sqrt{2}(e_2^{\mathrm{T}}-e_{30}^{\mathrm{T}})\quad 2(e_2^{\mathrm{T}}+2e_{30}^{\mathrm{T}}-6e_{32}^{\mathrm{T}})]^{\mathrm{T}},\rho_{10}=[\sqrt{2}(e_4^{\mathrm{T}}-e_{22}^{\mathrm{T}})\quad 2(e_4^{\mathrm{T}}+2e_{22}^{\mathrm{T}}-6e_{26}^{\mathrm{T}})]^{\mathrm{T}},$

$\rho_{11}=[\sqrt{2}(e_5^{\mathrm{T}}-e_{23}^{\mathrm{T}})\quad 2(e_5^{\mathrm{T}}+2e_{23}^{\mathrm{T}}-6e_{27}^{\mathrm{T}})]^{\mathrm{T}},\rho_{12}=[\sqrt{2}(e_6^{\mathrm{T}}-e_{24}^{\mathrm{T}})\quad 2(e_6^{\mathrm{T}}+2e_{24}^{\mathrm{T}}-6e_{28}^{\mathrm{T}})]^{\mathrm{T}},$

$\tilde{\boldsymbol{Q}}_{11}^j=\mathrm{diag}\{\tilde{\boldsymbol{Q}}_{11}^j,\tilde{\boldsymbol{Q}}_{11}^j,\tilde{\boldsymbol{Q}}_{11}^j\},j=1,2,\tilde{\boldsymbol{N}}_i=\mathrm{diag}\{\boldsymbol{N}_i,\boldsymbol{N}_i,\boldsymbol{N}_i\},i=1,2,$

$$\boldsymbol{\theta}_1(\Delta d_1(i))=\begin{bmatrix}\tilde{\boldsymbol{N}}_1 & \boldsymbol{X}_1\\ * & \tilde{\boldsymbol{N}}_1\end{bmatrix},\boldsymbol{\Xi}_1(\Delta d_1(i))=\begin{bmatrix}\tilde{\boldsymbol{Q}}_{11}^1 & \boldsymbol{X}_3\\ * & \tilde{\boldsymbol{Q}}_{11}^1\end{bmatrix},\boldsymbol{\Xi}_2(\Delta d_1(i))=\begin{bmatrix}\tilde{\boldsymbol{Q}}_{11}^2 & \boldsymbol{X}_4\\ * & \tilde{\boldsymbol{Q}}_{11}^2\end{bmatrix},$$

$$\boldsymbol{\theta}_2(\Delta d_2(j))=\begin{bmatrix}\tilde{\boldsymbol{N}}_2 & \boldsymbol{X}_2\\ * & \tilde{\boldsymbol{N}}_2\end{bmatrix},\boldsymbol{\Xi}_3(\Delta d_2(j))=\begin{bmatrix}\tilde{\boldsymbol{Q}}_{11}^1 & \boldsymbol{X}_5\\ * & \tilde{\boldsymbol{Q}}_{11}^1\end{bmatrix},\boldsymbol{\Xi}_4(\Delta d_2(j))=\begin{bmatrix}\tilde{\boldsymbol{Q}}_{11}^2 & \boldsymbol{X}_6\\ * & \tilde{\boldsymbol{Q}}_{11}^2\end{bmatrix}.$$

以下定理是本文推导出的 2-D 离散时变时滞系统(1)的新稳定性准则.

定理 1　系统(1)是渐近稳定的,如果存在 4 个 $4n\times4n$ 矩阵 $\boldsymbol{P}_{11}^1(\Delta d_1(i))>0,\boldsymbol{P}_{11}^2(\Delta d_1(i))>0,$ $\boldsymbol{P}_{11}^1(\Delta d_2(j))>0,\boldsymbol{P}_{11}^2(\Delta d_2(j))>0$,1 个 $4n\times2n$ 矩阵 $\boldsymbol{P}_{12}$,1 个 $2n\times2n$ 矩阵 $\boldsymbol{P}_{22}>0$,5 个 $n\times n$ 对称矩阵 $\boldsymbol{Q}_{11}^1(\Delta d_1(i)),\boldsymbol{Q}_{11}^2(\Delta d_1(i)),\boldsymbol{Q}_{11}^1(\Delta d_2(j)),\boldsymbol{Q}_{11}^2(\Delta d_2(j))$ 和 $\boldsymbol{Q}_{10}$,1 个 $n\times2n$ 矩阵 $\boldsymbol{Q}_{12}$,1 个 $2n\times2n$ 矩阵满足 $\boldsymbol{Q}_{22}>0$,6 个 $n\times n$ 矩阵 $\boldsymbol{Q}_3>0,\boldsymbol{Q}_4>0,\boldsymbol{S}_1>0,\boldsymbol{S}_2>0,\boldsymbol{R}_1>0$ 以及 $\boldsymbol{R}_2>0$,6 个 $3n\times3n$ 对称矩阵 $\boldsymbol{X}_i$, $i=1,2,\cdots,6$, 使得以下 LMIs 成立:

$$\begin{cases}\begin{bmatrix}\bar{d}_1\boldsymbol{P}_{11}^i & \boldsymbol{P}_{12}\\ * & \boldsymbol{P}_{22}\end{bmatrix}>0,\begin{bmatrix}\bar{d}_1\boldsymbol{Q}_{11}^i\boldsymbol{Q}_{10} & \boldsymbol{Q}_{12}\\ * & \boldsymbol{Q}_{22}\end{bmatrix}>0,\\ \begin{bmatrix}\bar{d}_2\boldsymbol{P}_{11}^i & \boldsymbol{P}_{12}\\ * & \boldsymbol{P}_{22}\end{bmatrix}>0,\begin{bmatrix}\bar{d}_2\boldsymbol{Q}_{11}^i\boldsymbol{Q}_{10} & \boldsymbol{Q}_{12}\\ * & \boldsymbol{Q}_{22}\end{bmatrix}>0,\end{cases} \quad i=1,2, \tag{9}$$

$$\begin{cases}N_1^-:=\boldsymbol{Q}_3-(\mu_1\boldsymbol{Q}_{11}^1(\Delta d_1(i))+\mu_1\boldsymbol{Q}_{11}^2(\Delta d_1(j)))>0,\\ N_2^-:=\boldsymbol{Q}_4-(\mu_2\boldsymbol{Q}_{11}^1(\Delta d_2(i))+\mu_2\boldsymbol{Q}_{11}^2(\Delta d_2(j)))>0,\end{cases} \tag{10}$$

$$\begin{cases}\begin{bmatrix}\widetilde{\boldsymbol{N}}_1+\widetilde{\boldsymbol{R}}_1 & \boldsymbol{X}_1\\ * & \widetilde{\boldsymbol{N}}_1\end{bmatrix}\geq 0,\begin{bmatrix}\widetilde{\boldsymbol{Q}}_{11}^1 & \boldsymbol{X}_3\\ * & \widetilde{\boldsymbol{Q}}_{11}^1\end{bmatrix}\geq 0,\begin{bmatrix}\widetilde{\boldsymbol{Q}}_{11}^1 & \boldsymbol{X}_5\\ * & \widetilde{\boldsymbol{Q}}_{11}^1\end{bmatrix}\geq 0,\\ \begin{bmatrix}\widetilde{\boldsymbol{N}}_2+\widetilde{\boldsymbol{R}}_2 & \boldsymbol{X}_2\\ * & \widetilde{\boldsymbol{N}}_2\end{bmatrix}\geq 0,\begin{bmatrix}\widetilde{\boldsymbol{Q}}_{11}^2 & \boldsymbol{X}_4\\ * & \widetilde{\boldsymbol{Q}}_{11}^2\end{bmatrix}\geq 0,\begin{bmatrix}\widetilde{\boldsymbol{Q}}_{11}^2 & \boldsymbol{X}_6\\ * & \widetilde{\boldsymbol{Q}}_{11}^2\end{bmatrix}\geq 0,\end{cases} \tag{11}$$

$$\Psi(\Delta d_1(i),\Delta d_2(j))<0,\forall(\Delta d_1(i),\Delta d_2(j))\in\{\mu_{1m},\mu_{1M}\}\times\{\mu_{2m},\mu_{2M}\}, \tag{12}$$

其中

$$\Psi(\Delta d_1(i),\Delta d_2(j))=\Psi_1(\Delta d_1(i),\Delta d_2(j))-\Psi_2(\Delta d_1(i),\Delta d_2(j)),$$

$$\begin{aligned}\Psi_1(\Delta d_1(i),\Delta d_2(j))=\boldsymbol{\xi}^{\mathrm{T}}\{&\Delta d_1(i)\textstyle\prod_0^{\mathrm{T}}(-\boldsymbol{P}_{11}^1+\boldsymbol{P}_{11}^2)\prod_0+\Delta d_2(j)\prod_3^{\mathrm{T}}(-\boldsymbol{P}_{11}^1+\boldsymbol{P}_{11}^2)\prod_3+\\ &\textstyle\prod_2^{\mathrm{T}}\boldsymbol{P}_1\prod_2+\bar{d}_1\Delta d_1(i)\rho_1^{\mathrm{T}}(\boldsymbol{Q}_{11}^1-\boldsymbol{Q}_{11}^2)\rho_1+\bar{d}_1\prod_6^{\mathrm{T}}\boldsymbol{Q}_1(d_1(i))\prod_6-\bar{d}_2\prod_{11}^{\mathrm{T}}\boldsymbol{Q}_2(d_2(j))\prod_{11}+\\ &\textstyle\prod_5^{\mathrm{T}}\boldsymbol{P}_2\prod_5+\bar{d}_2\Delta d_2(j)\rho_2^{\mathrm{T}}(\boldsymbol{Q}_{11}^1-\boldsymbol{Q}_{11}^2)\rho_2+\bar{d}_2\prod_{10}^{\mathrm{T}}\boldsymbol{Q}_2(d_2(j))\prod_{10}-\bar{d}_1\prod_7^{\mathrm{T}}\boldsymbol{Q}_1(d_1(i))\prod_7+\\ &\textstyle\bar{d}_1^2\prod_8^{\mathrm{T}}\boldsymbol{Q}_1(d_1(i))\prod_8+\bar{d}_2^2\rho_{14}^{\mathrm{T}}\boldsymbol{Q}_4\rho_{14}+\dfrac{d_{1M}(d_{1M}+1)}{2}\rho_{13}^{\mathrm{T}}\boldsymbol{S}_1P_{13}+\dfrac{d_1(d_1+1)}{2}e_{15}^{\mathrm{T}}\boldsymbol{R}_1e_{15}+\\ &\textstyle\bar{d}_2^2\prod_{12}^{\mathrm{T}}\boldsymbol{Q}_2(d_2(j))\prod_{12}+\bar{d}_1^2\rho_{12}^{\mathrm{T}}\boldsymbol{Q}_3\rho_{12}+\dfrac{d_{2M}(d_{2M}+1)}{2}\rho_{14}^{\mathrm{T}}\boldsymbol{S}_2\rho_{14}+\dfrac{d_2(d_2+1)}{2}e_{16}^{\mathrm{T}}\boldsymbol{R}_2e_{16}+\\ &\textstyle\mathrm{sym}(\prod_1^{\mathrm{T}}\boldsymbol{P}_1\prod_2+\prod_4^{\mathrm{T}}\boldsymbol{P}_2\prod_5+\bar{d}_1\prod_9^{\mathrm{T}}\boldsymbol{Q}(d_1(i))\prod_8+\bar{d}_2\prod_{13}^{\mathrm{T}}\boldsymbol{Q}(d_2(j))\prod_{13})\}\boldsymbol{\xi},\end{aligned}$$

$$\begin{aligned}\Psi_2(\Delta d_1(i),\Delta d_2(j))=&\rho_7^{\mathrm{T}}\widetilde{\boldsymbol{S}}_1\rho_7+\rho_8^{\mathrm{T}}\widetilde{\boldsymbol{S}}_2\rho_8+\rho_9^{\mathrm{T}}\widetilde{\boldsymbol{R}}_1\rho_9+\rho_{10}^{\mathrm{T}}\widetilde{\boldsymbol{R}}_2\rho_{10}+\rho_{11}^{\mathrm{T}}\widetilde{\boldsymbol{R}}_1\rho_{11}+\rho_{12}^{\mathrm{T}}\widetilde{\boldsymbol{S}}_2\rho_{12}+\\ &\rho_{15}^{\mathrm{T}}(\boldsymbol{Q}_1(\Delta d_1(i))+(\mu_1-\Delta d_1(i))\boldsymbol{\Xi}_1(\Delta d_1(i))+(\mu_1+\Delta d_1(i))\boldsymbol{\Xi}_2(\Delta d_1(i))\rho_{15}+\\ &\rho_{16}^{\mathrm{T}}(\boldsymbol{Q}_2(\Delta d_2(i))+(\mu_2-\Delta d_2(i))\boldsymbol{\Xi}_3(\Delta d_2(i))+(\mu_2+\Delta d_2(i))\boldsymbol{\Xi}_4(\Delta d_2(i))\rho_{16}.\end{aligned}$$

证明 考虑 LKF(6)与 4 个时滞相关矩阵 $\boldsymbol{P}_1(d_1(i))$,$\boldsymbol{P}_2(d_2(j))$,$\boldsymbol{Q}_1(d_1(i))$ 和 $\boldsymbol{Q}_2(d_2(j))$ 被表示如式(7)和(8).通过计算,矩阵 $\boldsymbol{P}_1(d_1(i))$,$\boldsymbol{P}_2(d_2(j))$,$\boldsymbol{P}_1(d_1(i+1))$,$\boldsymbol{P}_2(d_2(j+1))$,$\boldsymbol{Q}_1(d_1(i))$,$\boldsymbol{Q}_2(d_2(j))$,$\boldsymbol{Q}_1(d_1(i+1))$ 和 $\boldsymbol{Q}_2(d_2(j+1))$ 可重新表示如下:

$$\begin{cases}\boldsymbol{P}_1(d_1(i))=\dfrac{1}{\bar{d}_1}\left\{(d_{1M}-d_1(i))\begin{bmatrix}\bar{d}_1\boldsymbol{P}_{11}^1(i,0) & \boldsymbol{P}_{12}\\ * & \boldsymbol{P}_{22}\end{bmatrix}+(d_1(i)-d_{1m})\begin{bmatrix}\bar{d}_1\boldsymbol{P}_{11}^2(i,0) & \boldsymbol{P}_{12}\\ * & \boldsymbol{P}_{22}\end{bmatrix}\right\},\\ \boldsymbol{P}_2(d_2(j))=\dfrac{1}{\bar{d}_2}\left\{(d_{2M}-d_2(i))\begin{bmatrix}\bar{d}_2\boldsymbol{P}_{11}^1(0,j) & \boldsymbol{P}_{12}\\ * & \boldsymbol{P}_{22}\end{bmatrix}+(d_2(j)-d_{2m})\begin{bmatrix}\bar{d}_2\boldsymbol{P}_{11}^2(0,j) & \boldsymbol{P}_{12}\\ * & \boldsymbol{P}_{22}\end{bmatrix}\right\},\\ \boldsymbol{Q}_1(d_1(i))=\dfrac{1}{\bar{d}_1}\left\{(d_{1M}-d_1(i))\begin{bmatrix}\bar{d}_1\boldsymbol{Q}_{11}^1(i,0)+\boldsymbol{Q}_{10} & \boldsymbol{Q}_{12}\\ * & \boldsymbol{Q}_{22}\end{bmatrix}+(d_1(i)-d_{1m})\begin{bmatrix}\bar{d}_1\boldsymbol{Q}_{11}^2(i,0)+\boldsymbol{Q}_{10} & \boldsymbol{Q}_{12}\\ * & \boldsymbol{Q}_{22}\end{bmatrix}\right\},\\ \boldsymbol{Q}_2(d_2(j))=\dfrac{1}{\bar{d}_2}\left\{(d_{2M}-d_2(j))\begin{bmatrix}\bar{d}_2\boldsymbol{Q}_{11}^1(0,j)+\boldsymbol{Q}_{10} & \boldsymbol{Q}_{12}\\ * & \boldsymbol{Q}_{22}\end{bmatrix}+(d_2(j)-d_{2m})\begin{bmatrix}\bar{d}_2\boldsymbol{Q}_{11}^2(0,j)+\boldsymbol{Q}_{10} & \boldsymbol{Q}_{12}\\ * & \boldsymbol{Q}_{22}\end{bmatrix}\right\},\end{cases} \tag{13}$$

$$\begin{cases} \boldsymbol{P}_1(d_1(i+1)) = \begin{bmatrix} \Delta d_1(i)(-\boldsymbol{P}_{11}^1(i,0)+\boldsymbol{P}_{11}^2(i,0)) & 0_{4n\times 2n} \\ 0_{4n\times 2n} & 0_{2n\times 2n} \end{bmatrix} + \boldsymbol{P}_1(d_1(i)), \\ \boldsymbol{P}_2(d_2(j+1)) = \begin{bmatrix} \Delta d_2(j)(-\boldsymbol{P}_{11}^1(0,j)+\boldsymbol{P}_{11}^2(0,j)) & 0_{4n\times 2n} \\ 0_{4n\times 2n} & 0_{2n\times 2n} \end{bmatrix} + \boldsymbol{P}_2(d_2(j)), \\ \boldsymbol{Q}_1(d_1(i+1)) = \begin{bmatrix} \Delta d_1(i)(\boldsymbol{Q}_{11}^1(i,0)-\boldsymbol{Q}_{11}^2(i,0)) & 0_{n\times 2n} \\ 0_{2n\times n} & 0_{2n\times 2n} \end{bmatrix} + \boldsymbol{Q}_1(d_1(i)), \\ \boldsymbol{Q}_2(d_2(j+1)) = \begin{bmatrix} \Delta d_2(j)(\boldsymbol{Q}_{11}^1(0,j)-\boldsymbol{Q}_{11}^2(0,j)) & 0_{n\times 2n} \\ 0_{2n\times n} & 0_{2n\times 2n} \end{bmatrix} + \boldsymbol{Q}_2(d_2(j)), \end{cases} \tag{14}$$

通过使用式(14)、(20)和(21)，得到了 $\boldsymbol{P}(d_1(i))>0$，$\boldsymbol{P}(d_2(j))>0$，$\boldsymbol{Q}_1(d_1(i))>0$ 和 $\boldsymbol{Q}_2(d_2(j))>0$，这意味着存在一个正数 $\lambda_1>0$，能够使得

$$V \geq \lambda_1 \| x(i,j) \|^2 > 0, i=1,2,\cdots, j=1,2,\cdots, \tag{15}$$

根据 V 作前向差分，$\Delta V(i,j)=V(i+1,j+1)-V(i,j)$，可得到：

$$\Delta\bar{V}_1 = \boldsymbol{\zeta}^{\mathrm{T}}\{\Delta d_1(i)\prod\nolimits_0^{\mathrm{T}}(-\boldsymbol{P}_{11}^1(i,0)+\boldsymbol{P}_{11}^2(i,0))\prod\nolimits_0 + \prod\nolimits_2^{\mathrm{T}}\boldsymbol{P}_1\prod\nolimits_2 + \mathrm{sym}(\prod\nolimits_1^{\mathrm{T}}\boldsymbol{P}_1\prod\nolimits_2)\}\boldsymbol{\zeta},$$

$$\Delta\bar{V}_2 = \boldsymbol{\zeta}^{\mathrm{T}}\{\bar{d}_1\Delta d_1(i)\rho_1^{\mathrm{T}}(\boldsymbol{Q}_{11}^1-\boldsymbol{Q}_{11}^2)\rho_1 + \bar{d}_1\prod\nolimits_6^{\mathrm{T}}\boldsymbol{Q}_1(d_1(i))\prod\nolimits_6 - \bar{d}_1\prod\nolimits_7^{\mathrm{T}}\boldsymbol{Q}_1(d_1(i))\prod\nolimits_7 +$$

$$\bar{d}_1^2\prod\nolimits_8^{\mathrm{T}}\boldsymbol{Q}_1(d_1(i))\prod\nolimits_8 + \mathrm{sym}(\bar{d}_1\prod\nolimits_9^{\mathrm{T}}\boldsymbol{Q}_1(d_1(i))\prod\nolimits_8)\}\boldsymbol{\zeta} +$$

$$\bar{d}_1\Delta d_1(i)\sum_{s=-d_{1M}+1}^{-d_{1m}} y^{\mathrm{T}}(s,1)(\boldsymbol{Q}_{11}^1(i,0)-\boldsymbol{Q}_{11}^2(i,0))y(s,1),$$

$$\Delta\bar{V}_3 = \bar{d}_1^2 y^{\mathrm{T}}(0,1)\boldsymbol{Q}_3 y(0,1) - \bar{d}_1\sum_{s=-d_{1M}}^{-d_{1m}-1} y^{\mathrm{T}}(s,1)\boldsymbol{Q}_3 y(s,1),$$

$$\Delta\bar{V}_4 = \frac{d_{1M}(d_{1M}+1)}{2}y^{\mathrm{T}}(0,1)\boldsymbol{S}_1 y(0,1) - \sum_{s=-d_{1M}}^{-1}\sum_{u=s}^{-1} y^{\mathrm{T}}(u,1)\boldsymbol{S}_1 y(u,1),$$

$$\Delta\bar{V}_5 = \frac{\bar{d}_1(\bar{d}_1+1)}{2}y^{\mathrm{T}}(0,1)\boldsymbol{R}_1 y(0,1) - \sum_{s=-d_{1M}}^{-d_{1(i)}-1}\sum_{u=s}^{-d_{1(i)}-1} y^{\mathrm{T}}(u,1)\boldsymbol{R}_1 y(u,1) -$$

$$(d_{1M}-d_1(i))\sum_{u=-d_1(i)}^{-d_{1m}-1} y^{\mathrm{T}}(u,1)\boldsymbol{R}_1 y(u,1) - \sum_{s=-d_{1(i)}}^{-d_{1m}-1}\sum_{u=s}^{-d_{1m}-1} y^{\mathrm{T}}(u,1)\boldsymbol{R}_1 y(u,1).$$

使用类似的方法处理 $\Delta\hat{V}$，得到：

$$\Delta\hat{V}_1 = \boldsymbol{\zeta}^{\mathrm{T}}\{\Delta d_2(j)\prod\nolimits_3^{\mathrm{T}}(-\boldsymbol{P}_{11}^1(0,j)+\boldsymbol{P}_{11}^2(0,j))\prod\nolimits_3 + \prod\nolimits_5^{\mathrm{T}}\boldsymbol{P}_2\prod\nolimits_5 + \mathrm{sym}(\prod\nolimits_4^{\mathrm{T}}\boldsymbol{P}_2\prod\nolimits_5)\}\boldsymbol{\zeta},$$

$$\Delta\hat{V}_2 = \boldsymbol{\zeta}^{\mathrm{T}}\{\bar{d}_2\Delta d_2(j)\rho_2^{\mathrm{T}}(\boldsymbol{Q}_{11}^1-\boldsymbol{Q}_{11}^2)\rho_2 + \bar{d}_2\prod\nolimits_{10}^{\mathrm{T}}\boldsymbol{Q}_2\prod\nolimits_{10} - \bar{d}_2\prod\nolimits_{11}^{\mathrm{T}}\boldsymbol{Q}_2\prod\nolimits_{11} + \bar{d}_2^2\prod\nolimits_{12}^{\mathrm{T}}\boldsymbol{Q}_2\prod\nolimits_{12} +$$

$$\mathrm{sym}(\bar{d}_2\prod\nolimits_{13}^{\mathrm{T}}\boldsymbol{Q}_2\prod\nolimits_{13})\}\boldsymbol{\zeta} + \bar{d}_2\Delta d_2(j)\sum_{s=-d_{1M}+1}^{-d_{1m}-1} y^{\mathrm{T}}(1,s)(\boldsymbol{Q}_{11}^1(0,j)-\boldsymbol{Q}_{11}^2(0,j))y(1,s),$$

$$\Delta\hat{V}_3 = \bar{d}_2^2 y^{\mathrm{T}}(1,0)\boldsymbol{Q}_4 y(1,0) - \bar{d}_2\sum_{s=-d_{2M}}^{-d_{2m}-1} y^{\mathrm{T}}(1,s)\boldsymbol{Q}_4 y(1,s),$$

$$\Delta\hat{V}_4 = \frac{d_{2M}(d_{2M}+1)}{2}y^{\mathrm{T}}(1,0)\boldsymbol{S}_2 y(1,0) - \sum_{s=-d_{2M}}^{-1}\sum_{u=s}^{-1} y^{\mathrm{T}}(1,u)\boldsymbol{S}_2 y(1,u),$$

$$\Delta\hat{V}_5 = \frac{\bar{d}_2(\bar{d}_2+1)}{2}y^{\mathrm{T}}(1,0)\boldsymbol{R}_2 y(1,0) - \sum_{s=-d_{2M}}^{-d_{2(j)}-1}\sum_{u=s}^{-d_{2(j)}-1} y^{\mathrm{T}}(1,u)\boldsymbol{R}_2 y(1,u) -$$

$$(d_{2M}-d_2(j))\sum_{u=-d_2(j)}^{-d_{2m}-1} y^{\mathrm{T}}(1,u)\boldsymbol{R}_2 y(1,u) - \sum_{s=-d_{2(j)}}^{-d_{2m}-1}\sum_{u=s}^{-d_{2m}-1} y^{\mathrm{T}}(1,u)\boldsymbol{R}_2 y(1,u).$$

根据 $N_1:=\boldsymbol{Q}_3-(\mu_1\boldsymbol{Q}_{11}^1+\mu_1\boldsymbol{Q}_{11}^2)$，$N_2:=\boldsymbol{Q}_4-(\mu_2\boldsymbol{Q}_{11}^1+\mu_2\boldsymbol{Q}_{11}^2)$，$\Delta\bar{V}_3$，$\Delta\hat{V}_3$ 中的第 2 个积分项就可表示为：

$$-\bar{d}_1\sum_{s=-d_{1M}}^{-d_{1m}-1}y^{\mathrm{T}}(s,1)\boldsymbol{Q}_3y(s,1)=-\bar{d}_1\sum_{s=-d_{1M}}^{-d_1(i)-1}y^{\mathrm{T}}(s,1)\boldsymbol{N}_1y(s,1)-\bar{d}_1\sum_{s=-d_1(i)}^{-d_{1m}-1}y^{\mathrm{T}}(s,1)\boldsymbol{N}_1y(s,1)-$$
$$\mu_1\bar{d}_1\sum_{s=-d_{1M}}^{-d_1(i)-1}y^{\mathrm{T}}(s,1)\boldsymbol{Q}_{11}^1y(s,1)-\mu_1\bar{d}_1\sum_{s=-d_1(i)}^{-d_{1m}-1}y^{\mathrm{T}}(s,1)\boldsymbol{Q}_{11}^1y(s,1)-$$
$$\mu_1\bar{d}_1\sum_{s=-d_{1M}}^{-d_1(i)-1}y^{\mathrm{T}}(s,1)\boldsymbol{Q}_{11}^2y(s,1)-\mu_1\bar{d}_1\sum_{s=-d_1(i)}^{-d_{1m}-1}y^{\mathrm{T}}(s,1)\boldsymbol{Q}_{11}^2y(s,1),$$

$$-\bar{d}_2\sum_{s=-d_{2M}}^{-d_{2m}-1}y^{\mathrm{T}}(1,s)\boldsymbol{Q}_4y(1,s)=-\bar{d}_2\sum_{s=-d_{2M}}^{-d_2(j)-1}y^{\mathrm{T}}(1,s)\boldsymbol{N}_2y(1,s)-\bar{d}_2\sum_{s=-d_2(j)}^{-d_{2m}-1}y^{\mathrm{T}}(1,s)\boldsymbol{N}_2y(1,s)-$$
$$\mu_2\bar{d}_2\sum_{s=-d_{2M}}^{-d_2(j)-1}y^{\mathrm{T}}(1,s)\boldsymbol{Q}_{11}^1y(1,s)-\mu_2\bar{d}_2\sum_{s=-d_2(j)}^{-d_{2m}-1}y^{\mathrm{T}}(1,s)\boldsymbol{Q}_{11}^1y(1,s)-$$
$$\mu_2\bar{d}_2\sum_{s=-d_{2M}}^{-d_2(j)-1}y^{\mathrm{T}}(1,s)\boldsymbol{Q}_{11}^2y(1,s)-\mu_2\bar{d}_2\sum_{s=-d_2(j)}^{-d_{2m}-1}y^{\mathrm{T}}(1,s)\boldsymbol{Q}_{11}^2y(1,s).$$

因此,就得到:

$$\Delta V=\boldsymbol{\zeta}^{\mathrm{T}}\boldsymbol{\Psi}_1[\Delta d_1(i),\Delta d_2(j)]\boldsymbol{\zeta}-\bar{d}_1\sum_{s=-d_{1M}}^{-d_1(i)-1}y^{\mathrm{T}}(s,1)\boldsymbol{N}_1y(s,1)-\bar{d}_2\sum_{s=-d_{2M}}^{-d_2(j)-1}y^{\mathrm{T}}(1,s)\boldsymbol{N}_2y(1,s)-$$
$$\bar{d}_1\sum_{s=-d_1(i)}^{-d_{1m}-1}y^{\mathrm{T}}(s,1)\boldsymbol{N}_1y(s,1)-\bar{d}_1(\mu_1-\Delta d_1(i))\sum_{s=-d_{1M}}^{-d_1(i)-1}y^{\mathrm{T}}(s,1)\boldsymbol{Q}_{11}^1y(1,s)-$$
$$\bar{d}_2\sum_{s=-d_2(j)}^{-d_{2m}-1}y^{\mathrm{T}}(1,s)\boldsymbol{N}_2y(1,s)-\bar{d}_1(\mu_1-\Delta d_1(i))\sum_{s=-d_1(i)}^{-d_{1m}-1}y^{\mathrm{T}}(s,1)\boldsymbol{Q}_{11}^1y(s,1)-$$
$$\bar{d}_1(\mu_1+\Delta d_1(i))\sum_{s=-d_{1M}}^{-d_1(i)-1}y^{\mathrm{T}}(s,1)\boldsymbol{Q}_{11}^2y(s,1)-(d_{2M}-d_2(j))\sum_{s=-d_2(j)}^{-d_{2m}-1}y^{\mathrm{T}}(1,s)\boldsymbol{R}_2y(1,s)-$$
$$\bar{d}_2(\mu_2-\Delta d_2(j))\sum_{s=-d_{2M}}^{-d_2(j)-1}y^{\mathrm{T}}(1,s)\boldsymbol{Q}_{11}^1y(1,s)-(d_{1M}-d_1(i))\sum_{s=-d_1(i)}^{-d_{1m}-1}y^{\mathrm{T}}(s,1)\boldsymbol{R}_1y(s,1)-$$
$$\bar{d}_2(\mu_2+\Delta d_2(j))\sum_{s=-d_2(j)}^{-d_{2m}-1}y^{\mathrm{T}}(1,s)\boldsymbol{Q}_{11}^2y(1,s)-\sum_{s=-d_{2M}}^{-1}\sum_{u=s}^{-1}y^{\mathrm{T}}(1,u)\boldsymbol{S}_2y(1,u)-$$
$$\bar{d}_2(\mu_2+\Delta d_2(j))\sum_{s=-d_{2M}}^{-d_2(j)-1}y^{\mathrm{T}}(1,s)\boldsymbol{Q}_{11}^2y(1,s)-\sum_{s=-d_{1M}}^{-1}\sum_{u=s}^{-1}y^{\mathrm{T}}(u,1)\boldsymbol{S}_1y(u,1)-$$
$$\sum_{s=-d_{1M}}^{-d_1(i)-1}\sum_{u=s}^{-d_1(i)-1}y^{\mathrm{T}}(u,1)\boldsymbol{R}_1y(u,1)-\bar{d}_1(\mu_1+\Delta d_1(i))\sum_{s=-d_1(i)}^{-d_{1m}-1}y^{\mathrm{T}}(s,1)\boldsymbol{Q}_{11}^2y(s,1)-$$
$$\sum_{s=-d_{2M}}^{-d_2(j)-1}\sum_{u=s}^{-d_2(j)-1}y^{\mathrm{T}}(1,u)\boldsymbol{R}_2y(1,u)-\bar{d}_2(\mu_2+\Delta d_2(j))\sum_{s=-d_2(j)}^{-d_{2m}-1}y^{\mathrm{T}}(1,s)\boldsymbol{Q}_{11}^1y(1,s)-$$
$$\sum_{s=-d_1(i)}^{-d_{1m}-1}\sum_{u=s}^{-d_{1m}-1}y^{\mathrm{T}}(u,1)\boldsymbol{R}_1y(u,1)-\sum_{s=-d_2(j)}^{-d_{2m}-1}\sum_{u=s}^{-d_{2m}-1}y^{\mathrm{T}}(1,u)\boldsymbol{R}_2y(1,u).$$

由于 $\boldsymbol{Q}_{11}^1(i,0)>0,\boldsymbol{Q}_{11}^2(i,0)>0,\boldsymbol{Q}_{11}^1(0,j)>0,\boldsymbol{S}_i>0,\boldsymbol{R}_i>0,i=1,2$ 和 $\boldsymbol{N}_1>0,\boldsymbol{N}_2>0$,通过使用 α_1, α_2,β_1,β_2, 可得到:

$$-\bar{d}_1\sum_{s=-d_1(i)}^{-d_{1m}-1}y^{\mathrm{T}}(s,1)\boldsymbol{N}_1y(s,1)\leq-\frac{1}{\alpha_1}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_5^{\mathrm{T}}\widetilde{\boldsymbol{N}}_1\rho_5\}\boldsymbol{\zeta},-\bar{d}_2\sum_{s=-d_2(j)}^{-d_{2m}-1}y^{\mathrm{T}}(1,s)\boldsymbol{N}_2y(1,s)\leq-\frac{1}{\alpha_2}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_6^{\mathrm{T}}\widetilde{\boldsymbol{N}}_2\rho_6\}\boldsymbol{\zeta},$$
$$-\bar{d}_1\sum_{s=-d_{1M}}^{-d_1(i)-1}y^{\mathrm{T}}(s,1)\boldsymbol{N}_1y(s,1)\leq-\frac{1}{\beta_1}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_3^{\mathrm{T}}\widetilde{\boldsymbol{N}}_1\rho_3\}\boldsymbol{\zeta},-\bar{d}_2\sum_{s=-d_{2M}}^{-d_2(j)-1}y^{\mathrm{T}}(1,s)\boldsymbol{N}_2y(1,s)\leq-\frac{1}{\beta_2}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_4^{\mathrm{T}}\widetilde{\boldsymbol{N}}_2\rho_4\}\boldsymbol{\zeta},$$
$$\sum_{s=-d_{1M}}^{-1}\sum_{u=s}^{-1}y^{\mathrm{T}}(u,1)\boldsymbol{S}_1y(u,1)\leq\boldsymbol{\zeta}^{\mathrm{T}}\{-\rho_7^{\mathrm{T}}\widetilde{\boldsymbol{S}}_1\rho_7\}\boldsymbol{\zeta},\sum_{s=-d_{2M}}^{-1}\sum_{u=s}^{-1}y^{\mathrm{T}}(1,u)\boldsymbol{S}_1y(1,u)\leq\boldsymbol{\zeta}^{\mathrm{T}}\{-\rho_8^{\mathrm{T}}\widetilde{\boldsymbol{S}}_2\rho_8\}\boldsymbol{\zeta},$$

$$\sum_{s=-d_{1M}}^{-d_1(i)-1}\sum_{u=s}^{-d_1(i)-1} y^{\mathrm{T}}(u,1)\boldsymbol{R}_1 y(u,1)\le \boldsymbol{\zeta}^{\mathrm{T}}\{-\rho_9^{\mathrm{T}}\widetilde{\boldsymbol{R}}_1\rho_9\}\boldsymbol{\zeta},\ \sum_{s=-d_{2M}}^{-d_2(j)-1}\sum_{u=s}^{-d_2(j)-1} y^{\mathrm{T}}(1,u)\boldsymbol{R}_2 y(1,u)\le \boldsymbol{\zeta}^{\mathrm{T}}\{-\rho_{10}^{\mathrm{T}}\widetilde{\boldsymbol{R}}_2\rho_{10}\}\boldsymbol{\zeta},$$

$$\sum_{s=-d_1(i)}^{-d_{1m}-1}\sum_{u=s}^{-d_{1m}-1} y^{\mathrm{T}}(u,1)\boldsymbol{R}_1 y(u,1)\le \boldsymbol{\zeta}^{\mathrm{T}}\{-\rho_{11}^{\mathrm{T}}\widetilde{\boldsymbol{R}}_1\rho_{11}\}\boldsymbol{\zeta},\ \sum_{s=-d_2(j)}^{-d_{2m}-1}\sum_{u=s}^{-d_{2m}-1} y^{\mathrm{T}}(1,u)\boldsymbol{R}_2 y(1,u)\le \boldsymbol{\zeta}^{\mathrm{T}}\{-\rho_{12}^{\mathrm{T}}\widetilde{\boldsymbol{R}}_2\rho_{12}\}\boldsymbol{\zeta},$$

$$-(d_{1M}-d_1(i))\sum_{s=-d_1(i)}^{-d_{1m}-1} y^{\mathrm{T}}(s,1)\boldsymbol{R}_1 y(s,1)\le -(\frac{1}{\alpha_1}-1)\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_5^{\mathrm{T}}\widetilde{\boldsymbol{R}}_1\rho_5\}\boldsymbol{\zeta},$$

$$-(d_{2M}-d_2(j))\sum_{s=-d_2(j)}^{-d_{2m}-1} y^{\mathrm{T}}(1,s)\boldsymbol{R}_2 y(1,s)\le -(\frac{1}{\alpha_2}-1)\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_6^{\mathrm{T}}\widetilde{\boldsymbol{R}}_2\rho_6\}\boldsymbol{\zeta},$$

$$-\bar{d}_1(\mu_1-\Delta d_1(i))\sum_{s=-d_{1M}}^{-d_1(i)-1} y^{\mathrm{T}}(s,1)\boldsymbol{Q}_{11}^1 y(s,1)\le -(\mu_1-\Delta d_1(i))\frac{1}{\beta_1}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_3^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^1\rho_3\}\boldsymbol{\zeta},$$

$$-\bar{d}_1(\mu_1+\Delta d_1(i))\sum_{s=-d_{1M}}^{-d_1(i)-1} y^{\mathrm{T}}(s,1)\boldsymbol{Q}_{11}^2 y(s,1)\le -(\mu_1+\Delta d_1(i))\frac{1}{\beta_1}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_3^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^2\rho_3\}\boldsymbol{\zeta},$$

$$-\bar{d}_2(\mu_2-\Delta d_2(j))\sum_{s=-d_{2M}}^{-d_2(j)-1} y^{\mathrm{T}}(1,s)\boldsymbol{Q}_{11}^1 y(1,s)\le -(\mu_2-\Delta d_2(j))\frac{1}{\beta_2}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_4^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^1\rho_4\}\boldsymbol{\zeta},$$

$$-\bar{d}_1(\mu_1+\Delta d_1(i))\sum_{s=-d_1(i)}^{-d_{1m}-1} y^{\mathrm{T}}(s,1)\boldsymbol{Q}_{11}^1 y(s,1)\le -(\mu_1-\Delta d_1(i))\frac{1}{\alpha_1}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_5^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^1\rho_5\}\boldsymbol{\zeta},$$

$$-\bar{d}_1(\mu_1-\Delta d_1(i))\sum_{s=-d_1(i)}^{-d_{1m}-1} y^{\mathrm{T}}(s,1)\boldsymbol{Q}_{11}^2 y(s,1)\le -(\mu_1+\Delta d_1(i))\frac{1}{\alpha_1}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_5^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^2\rho_5\}\boldsymbol{\zeta},$$

$$-\bar{d}_2(\mu_2-\Delta d_2(j))\sum_{s=-d_2(j)}^{-d_{2m}-1} y^{\mathrm{T}}(1,s)\boldsymbol{Q}_{11}^1 y(1,s)\le -(\mu_2-\Delta d_2(j))\frac{1}{\alpha_2}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_6^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^1\rho_6\}\boldsymbol{\zeta},$$

$$-\bar{d}_2(\mu_2+\Delta d_2(j))\sum_{s=-d_2(j)}^{-d_{2m}-1} y^{\mathrm{T}}(1,s)\boldsymbol{Q}_{11}^2 y(1,s)\le -(\mu_2+\Delta d_2(j))\frac{1}{\alpha_2}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_6^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^2\rho_6\}\boldsymbol{\zeta},$$

$$-\bar{d}_2(\mu_2+\Delta d_2(j))\sum_{s=-d_{2M}}^{-d_2(j)-1} y^{\mathrm{T}}(1,s)\boldsymbol{Q}_{11}^2 y(1,s)\le -(\mu_2+\Delta d_2(j))\frac{1}{\beta_2}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_4^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^2\rho_4\}\boldsymbol{\zeta},$$

根据(10)和(11),通过使用引理 2,就可得到以下 6 个估计不等式:

$$\begin{aligned}
&-\frac{1}{\alpha_1}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_5^{\mathrm{T}}\widetilde{\boldsymbol{N}}_1\rho_5\}\boldsymbol{\zeta}-\frac{1}{\beta_1}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_3^{\mathrm{T}}\widetilde{\boldsymbol{N}}_1\rho_3\}\boldsymbol{\zeta}-(\frac{1}{\alpha_1}-1)\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_5^{\mathrm{T}}\widetilde{\boldsymbol{R}}_1\rho_5\}\boldsymbol{\zeta}\le\boldsymbol{\zeta}^{\mathrm{T}}\{-\rho_{15}^{\mathrm{T}}\boldsymbol{\theta}_1\rho_{15}\}\boldsymbol{\zeta},\\
&-\frac{1}{\alpha_2}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_6^{\mathrm{T}}\widetilde{\boldsymbol{N}}_2\rho_6\}\boldsymbol{\zeta}-\frac{1}{\beta_2}\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_4^{\mathrm{T}}\widetilde{\boldsymbol{N}}_2\rho_4\}\boldsymbol{\zeta}-(\frac{1}{\alpha_2}-1)\boldsymbol{\zeta}^{\mathrm{T}}\{\rho_6^{\mathrm{T}}\widetilde{\boldsymbol{R}}_2\rho_6\}\boldsymbol{\zeta}\le\boldsymbol{\zeta}^{\mathrm{T}}\{-\rho_{16}^{\mathrm{T}}\boldsymbol{\theta}_2\rho_{16}\}\boldsymbol{\zeta},\\
&-(\mu_1-\Delta d_1(i))\boldsymbol{\zeta}^{\mathrm{T}}\{\frac{1}{\alpha_1}\rho_5^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^1(i,0)\rho_5+\frac{1}{\beta_1}\rho_3^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^1(i,0)\rho_3\}\boldsymbol{\zeta}\le\boldsymbol{\zeta}^{\mathrm{T}}\{-\rho_{15}^{\mathrm{T}}\boldsymbol{\Xi}_1\rho_{15}\}\boldsymbol{\zeta},\\
&-(\mu_1+\Delta d_1(i))\boldsymbol{\zeta}^{\mathrm{T}}\{\frac{1}{\alpha_1}\rho_5^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^2(i,0)\rho_5+\frac{1}{\beta_1}\rho_3^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^2(i,0)\rho_3\}\boldsymbol{\zeta}\le\boldsymbol{\zeta}^{\mathrm{T}}\{-\rho_{15}^{\mathrm{T}}\boldsymbol{\Xi}_2\rho_{15}\}\boldsymbol{\zeta},\\
&-(\mu_2-\Delta d_2(j))\boldsymbol{\zeta}^{\mathrm{T}}\{\frac{1}{\alpha_2}\rho_6^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^1(0,j)\rho_6+\frac{1}{\beta_2}\rho_4^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^1(0,j)\rho_4\}\boldsymbol{\zeta}\le\boldsymbol{\zeta}^{\mathrm{T}}\{-\rho_{16}^{\mathrm{T}}\boldsymbol{\Xi}_3\rho_{16}\}\boldsymbol{\zeta},\\
&-(\mu_2+\Delta d_2(j))\boldsymbol{\zeta}^{\mathrm{T}}\{\frac{1}{\alpha_2}\rho_6^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^2(0,j)\rho_6+\frac{1}{\beta_2}\rho_4^{\mathrm{T}}\widetilde{\boldsymbol{Q}}_{11}^2(0,j)\rho_4\}\boldsymbol{\zeta}\le\boldsymbol{\zeta}^{\mathrm{T}}\{-\rho_{16}^{\mathrm{T}}\boldsymbol{\Xi}_4\rho_{16}\}\boldsymbol{\zeta},
\end{aligned}\tag{16}$$

综上所述,就获得 $V\le\boldsymbol{\zeta}^{\mathrm{T}}\boldsymbol{\Psi}(d_1(i),d_2(j))\boldsymbol{\zeta}$. 证明系统是渐近稳定的,证毕.

推论 1　对于 LKFs(6)不使用时滞相关矩阵的情况,通过设置 $\boldsymbol{P}_{11}^1=\boldsymbol{P}_{11}^2$ 和 $\boldsymbol{Q}_{11}^1=\boldsymbol{Q}_{11}^2=0$,即 4 个矩阵 $\boldsymbol{P}_1(d_1(i))$,$\boldsymbol{P}_2(d_2(j))$,$\boldsymbol{Q}_1(d_1(i))$ 和 $\boldsymbol{Q}_2(d_2(j))$ 被简化为 4 个常数矩阵 $\boldsymbol{P}_1$,$\boldsymbol{P}_2$,$\boldsymbol{Q}_1$ 和 $\boldsymbol{Q}_2$,根据定理 1,就得到不使用时滞相关矩阵的系统稳定性准则:系统是渐近稳定的,如果存在 2 个 $6n\times6n$ 矩阵 $\boldsymbol{P}_i=\begin{bmatrix}\bar{d}_i\boldsymbol{P}_{11}^1 & \boldsymbol{P}_{12}\\ * & \boldsymbol{P}_{22}\end{bmatrix}>0$, $i=1,2$, 2 个 $3n\times3n$ 矩阵 $\boldsymbol{Q}_i=\begin{bmatrix}\boldsymbol{Q}_{10} & \boldsymbol{Q}_{12}\\ * & \boldsymbol{Q}_{22}\end{bmatrix}>0$, $i=1,2$, 6 个 $n\times n$ 矩阵 $\boldsymbol{Q}_3>0$, $\boldsymbol{Q}_4>0$,

$S_1>0$,$S_2>0$,$R_1>0$,$R_2>0$,2 个 $3n\times 3n$ 对称矩阵 X_1,X_2,使得以下 LMIs(9) 和(10) 满足 $Q_{11}^1=Q_{11}^2=0$,并且 $Q_1(d_1(i))$,$Q_2(d_2(j))$ 被 Q_1,Q_2 替代,N_1,N_2 被 Q_3,Q_4 所替代.

注 1 当 μ_{1m},μ_{2m},μ_{1M},μ_{2M},d_{1m},d_{2m},d_{1M} 和 d_{2M} 的值固定时,定理 1 和推论 1 中的稳定性条件将更改为 LMIs.通过使用 MATLAB 工具最大化时变时滞的算法,对于固定值,就可以获得最大时变时滞的上界 d_{2M} 的值,即时滞范围.

3 数值算例

在本节中,提出了两个数值算例,一个选择没有使用时滞相关矩阵,另一个则使用,以验证本文所提出稳定性准则的有效性.

例 1 化学反应器、热交换器或管式炉中的热过程可以用以下具有时滞的局部可微分方程来表示[6]:

$$\frac{\partial T(x,t)}{\partial x}=\frac{\partial T(x,t)}{\partial t}-a(x,t,T(x,t))T(x,t)-b(x,t,T(x,t))T(x,t-\tau_1),$$

其中 $T(x,t)$ 是空间 x 和时间 t 处的温度,τ_t 是时滞,$a(\cdot)=a(x,t,T(x,t))$,$b(\cdot)=b(x,t,T(x,t))$ 是系数函数,具体取决于状态 $T(x,t)$. 倘若

$$T(i,j)=T(i\Delta x,j\Delta t),\frac{\partial T(x,t)}{\partial x}\approx\frac{T(i,j)-T(i-1,j)}{\Delta x},\frac{\partial T(x,t)}{\partial t}\approx\frac{T(i,j+1)-T(i,j)}{\Delta x}.$$

下列 2-D 线性模型就可以得到:

$$T(i,j+1)=(1-\frac{\Delta t}{\Delta x}-a_0\Delta t)T(i,j)+\frac{\Delta t}{\Delta x}T(i-1,j)-a_1\Delta tT(i,j-d_2)+b\Delta tu(i,j).$$

记 $x^{\mathrm{T}}(i,j)=[T^{\mathrm{T}}(i-1,j)T^{\mathrm{T}}(i,j)]$,这样 2-D FM 模型就可以转化为:$x(i+1,j+1)=A_1x(i,j+1)+A_2x(i+1,j)+A_{1d}x(i-d_1(i),j+1)+A_{2d}x(i+1,j-d_2(j))$,其中

$$A_1=\begin{bmatrix}0 & 0\\ \frac{\Delta t}{\Delta x} & 1-\frac{\Delta t}{\Delta x}-a_0\Delta t\end{bmatrix},A_2=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix},A_{1d}=\begin{bmatrix}0 & 0\\ 0 & -a_1\Delta t\end{bmatrix},A_{2d}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}.$$

令 $\Delta t=0.1$,$\Delta x=0.4$,$a_0=1$,$a_1=1.2$,$b=1$ 将上述参数代入矩阵. 考虑具有以下系统矩阵和参数的 2-D 离散时滞系统(1)[28](不使用时滞相关矩阵):

$$A_1=\begin{bmatrix}0 & 0\\ 0.25 & 0.65\end{bmatrix}A_2=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}A_{1d}=\begin{bmatrix}0 & 0\\ 0 & -0.12\end{bmatrix}A_{2d}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}. \tag{17}$$

当时滞 $d_2(j)$ 对于 $d_1(i)=6+5\sin(\pi i/2)$ 具有时变性时,文献[32] 中的系统对于 $0\leq d_2(j)\leq 13$ 是渐近稳定的,即 $d_2(j)$ 的上界远大于文献[6] 中给出的上界.此外,对于推论 1,系统在满足 $0\leq d_2(j)\leq 20$ 下仍然是渐近稳定的,这表明时滞上界大于文献[32]中给出的.表 1 中列出了允许的最大上界.从表 1 中,可以看到推论 1 提供了最大的允许上界.图 1 和图 2 显示了系统两个状态变量在任意(随机生成)边界条件下的轨迹.

图 1 和图 2 显示,随着 i 和 j 继续增加,状态振幅变小并最终接近原点,即具有系数矩阵(17)的系统(1)在时滞上界 $d_{2M}=48$ 下渐近稳定.

例 2 研究了具有以下参数的 2-D 系统(1)(使用时滞相关矩阵)[33]:

$$A_1=\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix}A_2=\begin{bmatrix}0.4 & 0\\ 0.2 & 0\end{bmatrix}A_{1d}=\begin{bmatrix}0.02 & 0\\ 0 & 0.01\end{bmatrix}A_{2d}=\begin{bmatrix}0.01 & 0\\ 0 & 0.01\end{bmatrix}. \tag{18}$$

假设时变时滞满足条件:$\mu_{1M}=-\mu_{1m}=\mu_{2M}=-\mu_{1M}=\mu_{1M}=\mu$ 和条件(2).将文献[29]中的结果与本文中的 LMIs 决策变量数目进行比较,由表 2,本文决策变量的数目小于文献[29]定理 1 中的数量.为了比较时滞范围,对比文献[27-29,34-35]的定理 1 以及本文结论可以找到最大允许上界 d_{2M},并在表 3 中列出,且有:1)本文定理 1 得到的最大允许上界大于推论 1 中的,这表明本文设计的在单项求和项中引入时滞相关矩阵对于推导稳定性准则非常有意义;2)本文定理 1 和推论 1 中获得的最大允许上界大于文献[29]中获得的,这证实了本文推导的结果优于其他参考文献获得的结果.综上,本文获得的结果比参考文献的保守性更小.

表 1　例 1 中对于给定 d_{1m}，d_{2m} 和 d_{1M} 得到的最大允许时滞上界 d_{2M}

Tab. 1　Given d_{1m}，d_{2m} and d_{1M}，the resulting maximum allowable time delay upper bound d_{2M} in example 1

文献	d_{1m}	d_{2m}	d_{1M}	d_{2M}
文献[32]	1	10	1	13
文献[6]	1	10	1	20
推论 1	1	10	1	48

表 2　例 2 中不同方法中的 LMIs 决策变量数目

Tab. 2　The number of LMIs decision variables in different methods in example 2

方法	决策变量数目
文献[29]（定理 1）	$108n^2+26n$
推论 1	$94n^2+6n$
定理 1	$88n^2+23n$

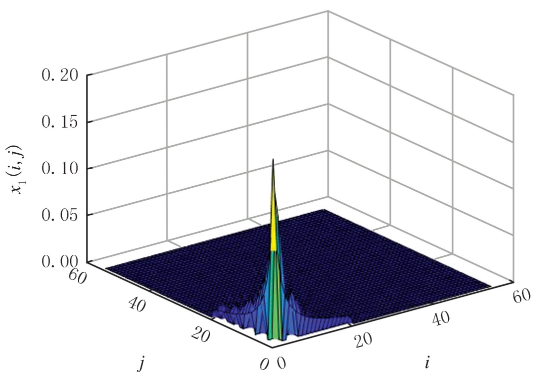

图1　系统(1)状态变量$x_1(i,j)$轨迹

Fig. 1　State $x_1(i,j)$ trajectory of the system(1)

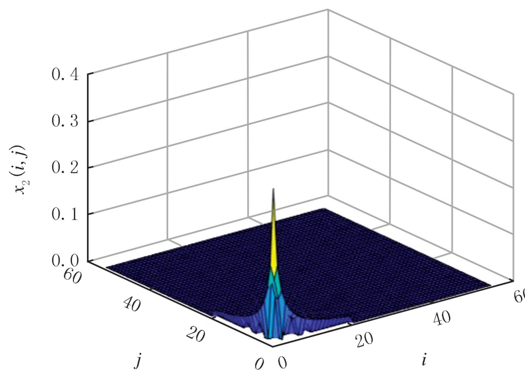

图2　系统(1)状态变量$x_2(i,j)$轨迹

Fig. 2　State $x_2(i,j)$ trajectory of the system(1)

表 3　例 2 中的允许时变时滞上限

Tab. 3　The upper limit of the allowable time delayin example 2

方法	d_{1m}	d_{1M}	d_{2m}	d_{2M}	方法	d_{1m}	d_{1M}	d_{2m}	d_{2M}
文献[27]	1	11	1	20	文献[29]	1	11	1	36
文献[28]	1	11	1	20	推论 1	1	11	1	56
文献[34]	1	11	1	20	定理 1	1	11	1	68
文献[35]	1	11	1	28					

系统的两个状态变量在任意(随机生成)边界条件下的轨迹如图 3 和图 4 所示.在初始状态下，状态波动一开始变化显著，随着 i 和 j 的增加，系统状态逐渐接近于零，因此本文给出的系统稳定性准则被证明是有效且有意义的.这也为以后设置控制器奠定了坚实的基础.

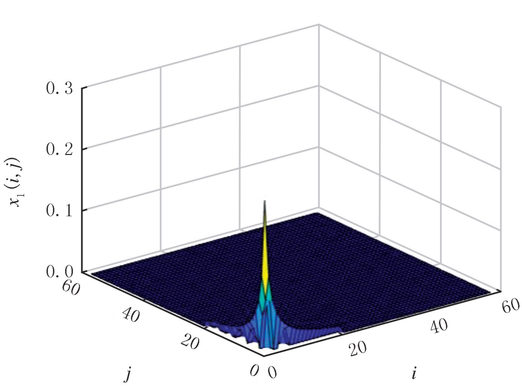

图3　系统(1)状态变量$x_1(i,j)$轨迹

Fig. 3　State $x_1(i,j)$ trajectory of the system(1)

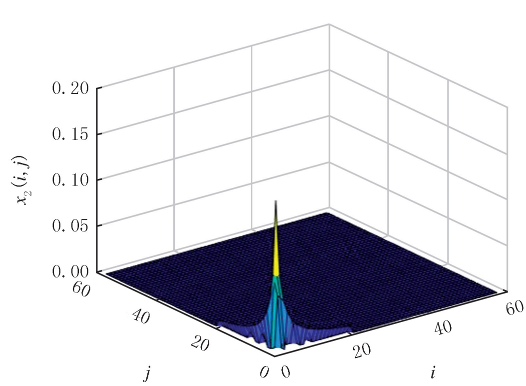

图4　系统(1)状态变量$x_2(i,j)$轨迹

Fig. 4　State $x_2(i,j)$ trajectory of the system(1)

4 结 论

本文创建具有三重求和项的 LKFs 同时考虑了时滞相关矩阵和增广向量矩阵，包含有关时滞上界和下界的更多信息.然后利用多重辅助函数不等式和互凸组合不等式获得了新的系统稳定性准则，扩大了时滞范围并降低了结果的保守性.同时本文结果减少了决策变量的数目，从而减轻了计算负担.最后，通过数值算例与现有结果进行对比，验证了本文所设计方法的有效性和优越性.本文设计的增广向量矩阵中可同时包含时滞的上界和下界，并以此用来估计双重求和项中的系统 LKFs 差分过程，进一步降低稳定性准则的保守性，但会加大差分难度和计算量，进而怎么处理 LKFs 差分产生的有限和项，值得学者们进一步探索.

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New delay-variation-dependent stability criterion for 2-D discrete systems with delays

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Abstract: The delay-variation-dependent stability problem for two-dimensional(2-D) discrete-time systems with delays is studied. The Lyapunov-Krasovskii functionals(LKFs) are constructed by using delay-dependent matrices in the quadratic and single-sum terms, respectively, considering more state information. It is also the first time that the augmented vector matrices in the single summation term have been studied the system stability. Meanwhile, the multiple auxiliary function inequality and reciprocally convex inequality suitable for 2-D systems are given to process LKFs differentiation so as to reduce the computational burden. Derive a less conservative stability criterion for 2-D discrete systems with time-varying delays. The effectiveness and superiority of the devised method is confirmed by two numerical examples.

Keywords: two-dimensional discrete systems; time-varying delays; Lyapunov-Krasovskii functionals; multiple auxiliary function inequality; reciprocally convex inequality

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